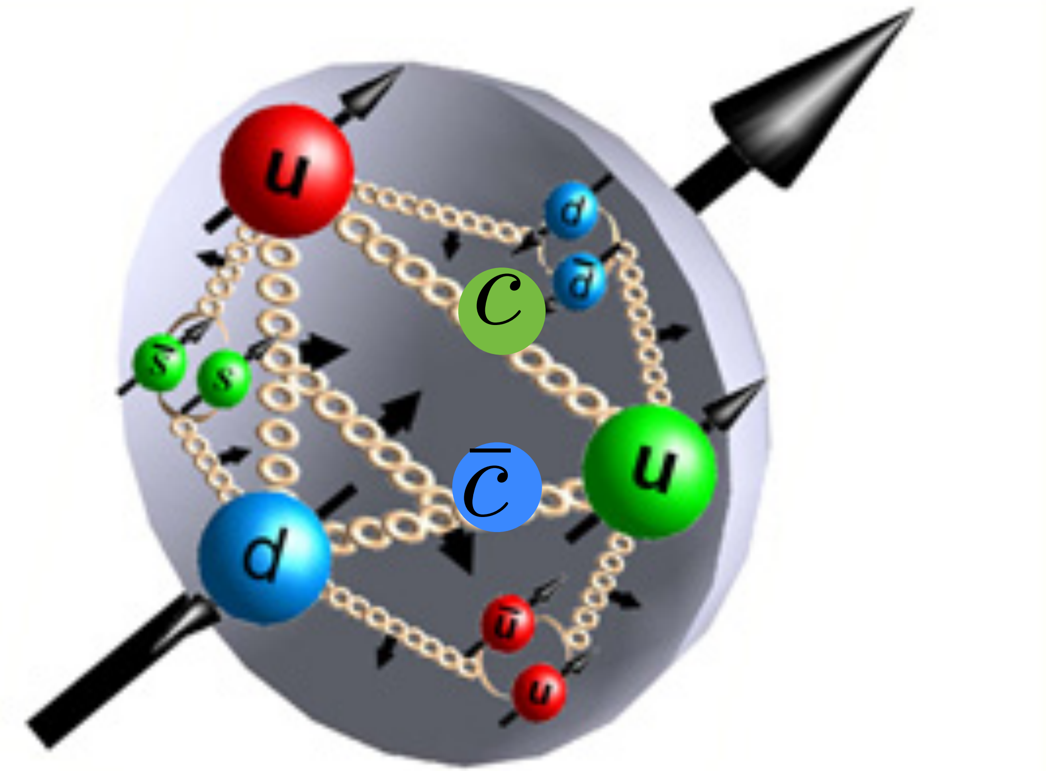
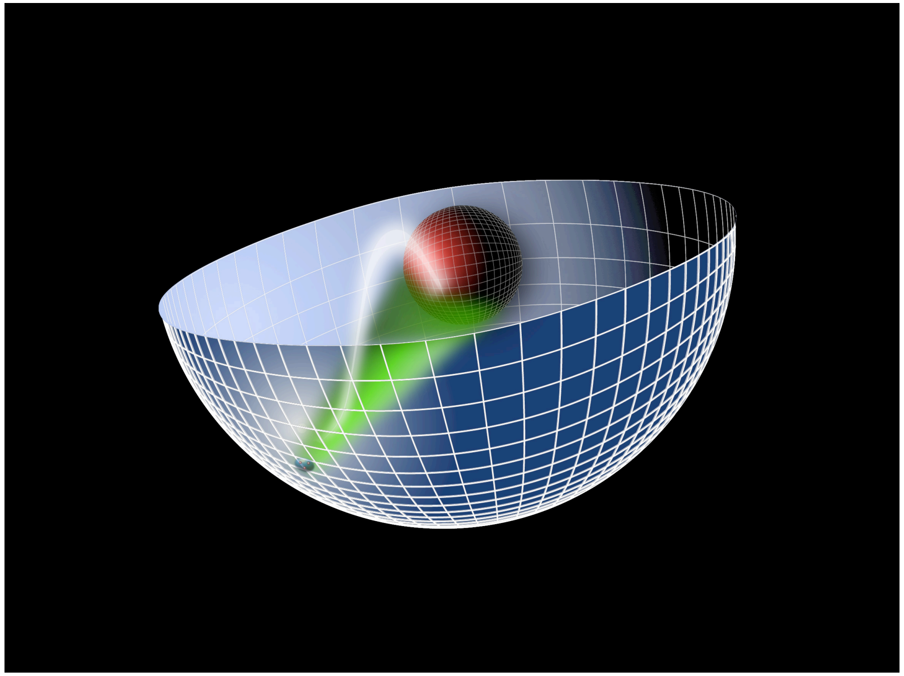
Novel World of Hadron Physics





Stan Brodsky

Colloquium
April 27, 2015

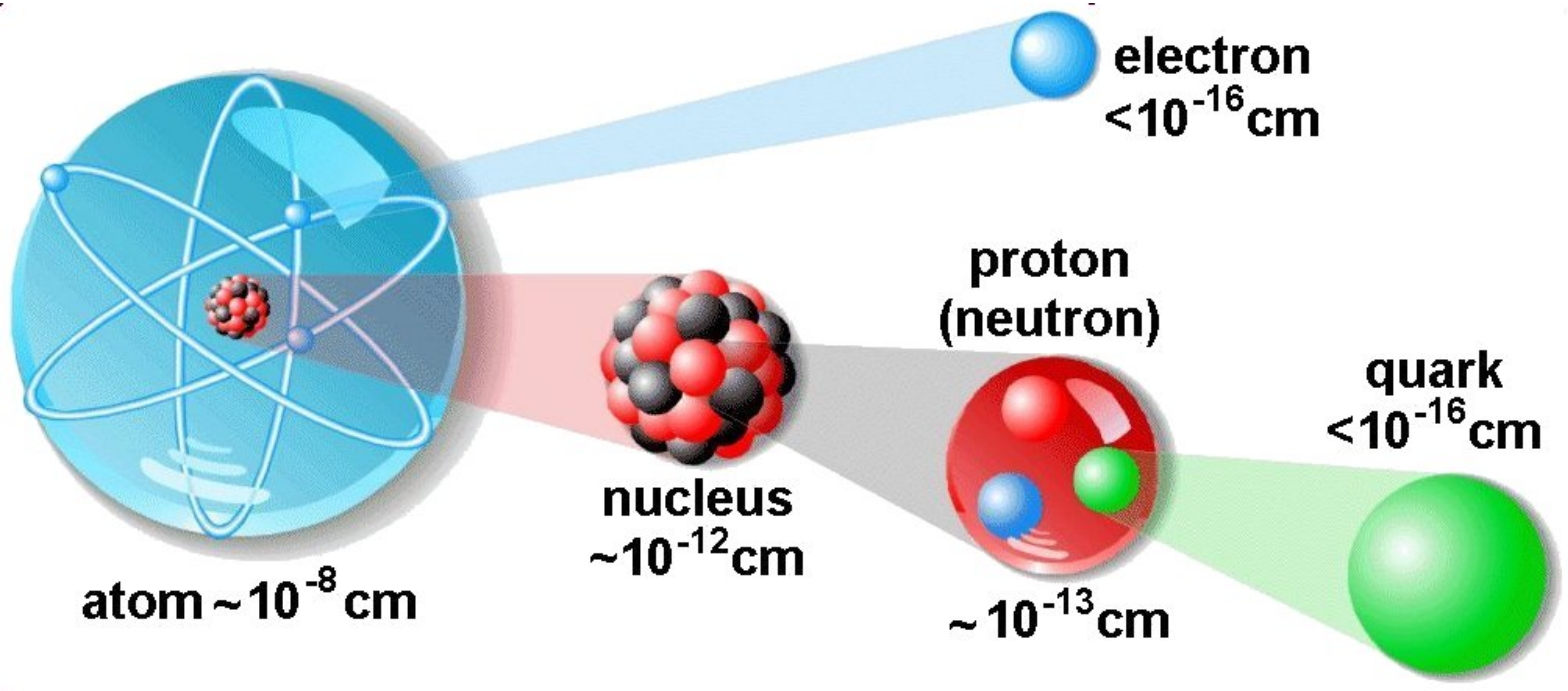
SLAC
NATIONAL ACCELERATOR LABORATORY

UNIVERSITY *of* VIRGINIA

Stanford University

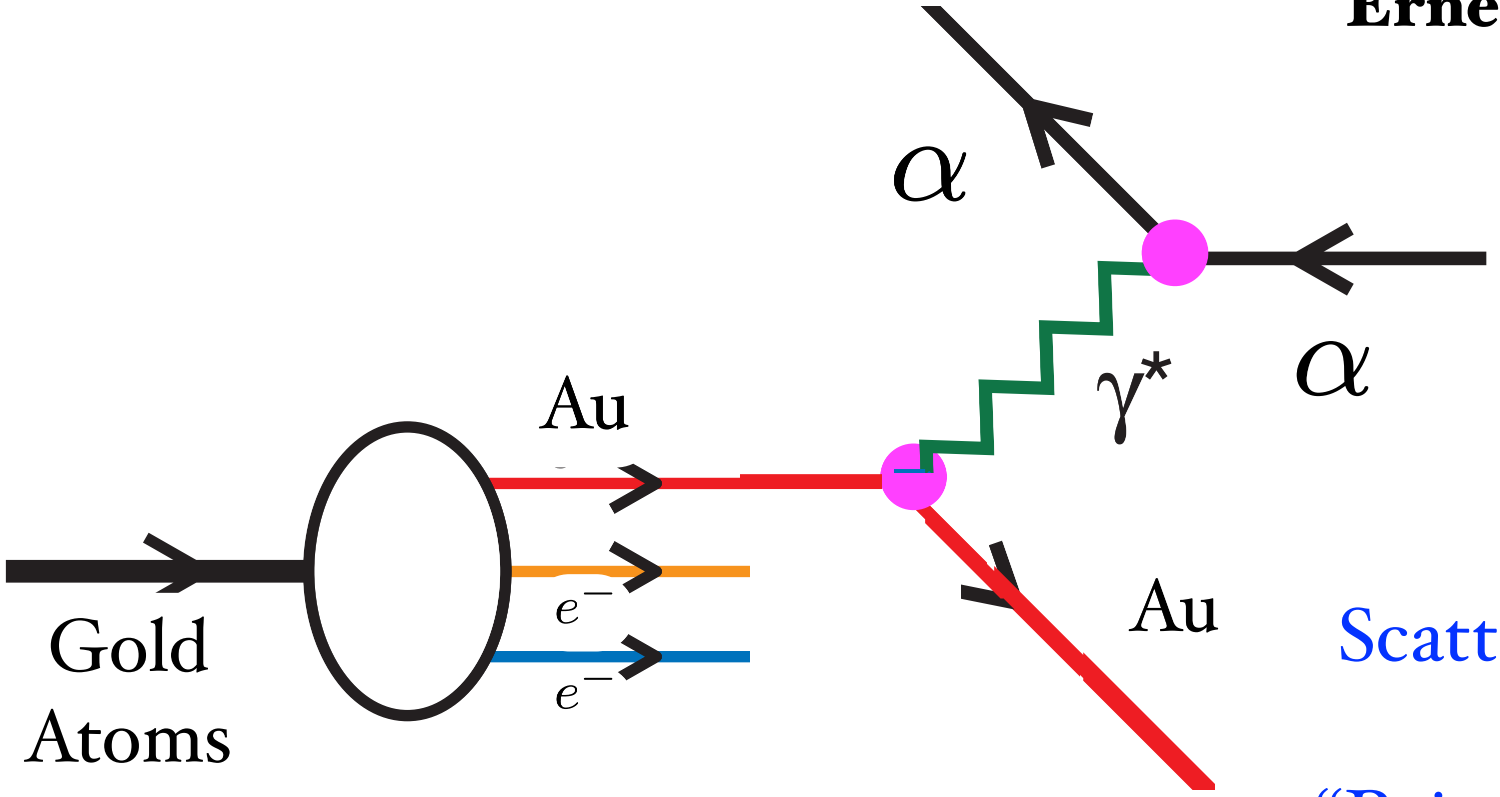
Goal of Science:

To understand the laws of physics and the fundamental composition of matter at the shortest possible distances.



First Evidence for Nuclear Structure of Atoms

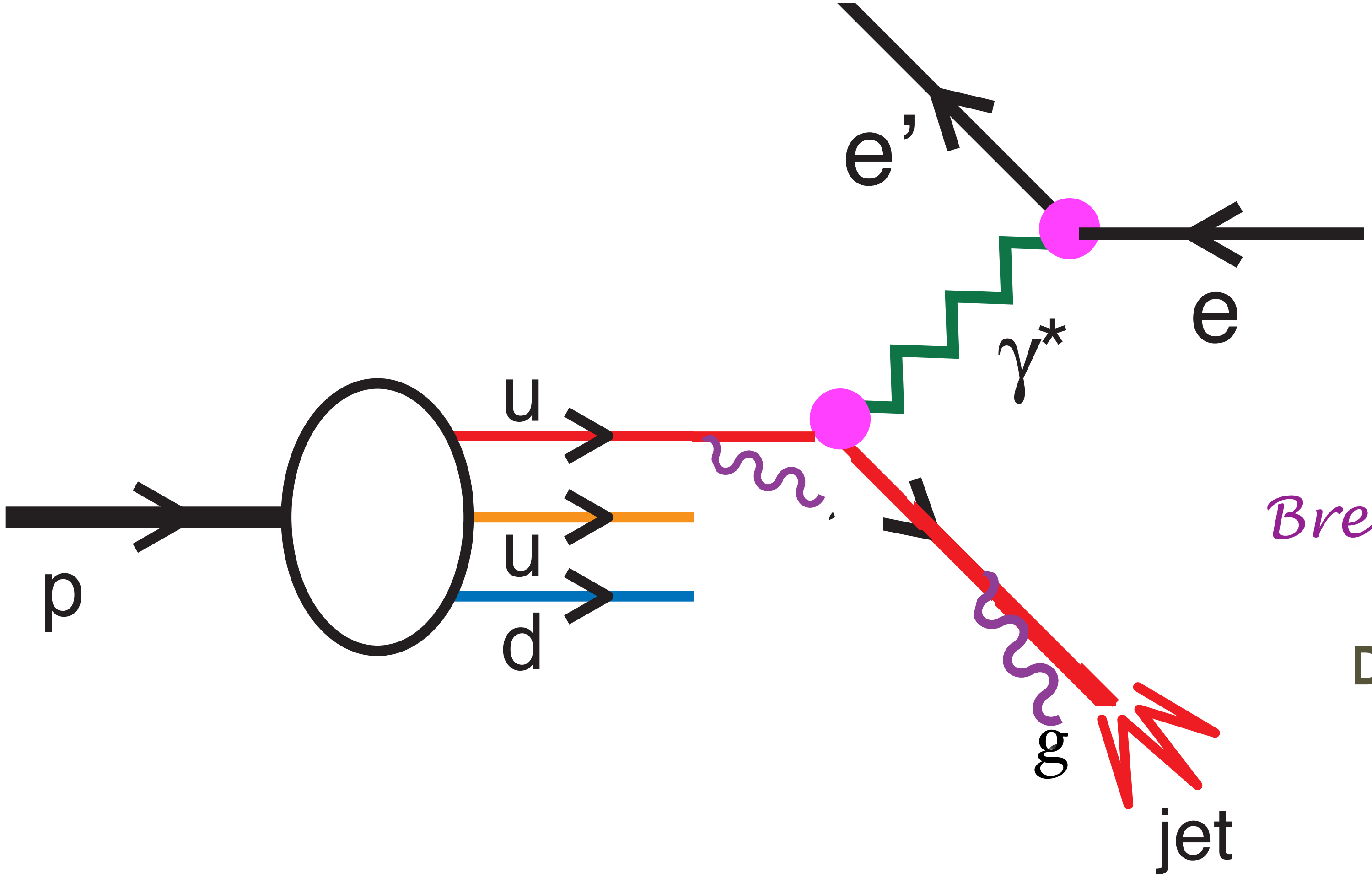
Ernest Rutherford
1911



Scattering at Large Angles!
"Point-like" Nucleus

Rutherford Scattering

First Evidence for Quark Structure of Matter

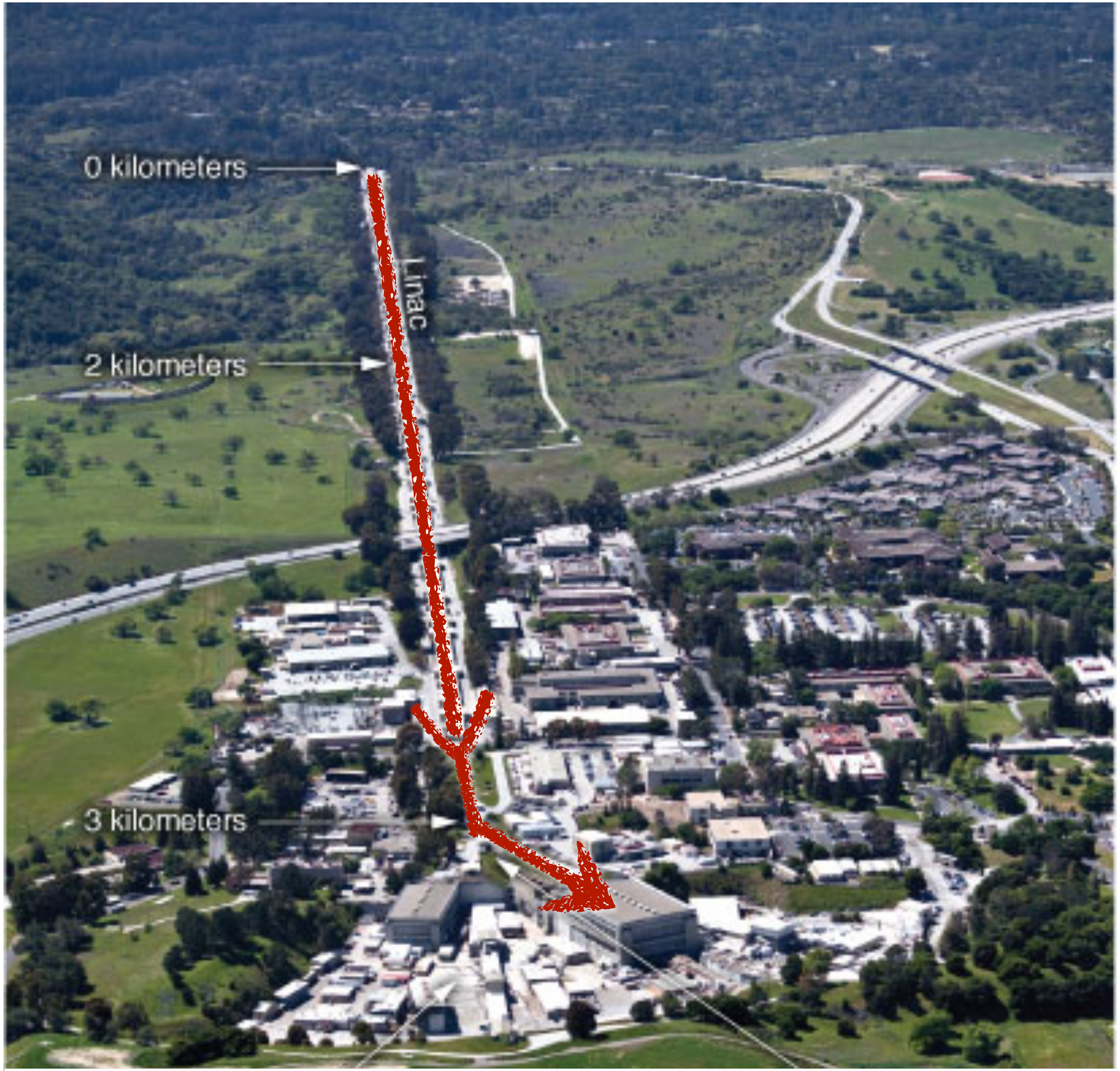


Gluonic Bremmstrahlung

DGLAP Evolution

Deep Inelastic Electron-Proton Scattering

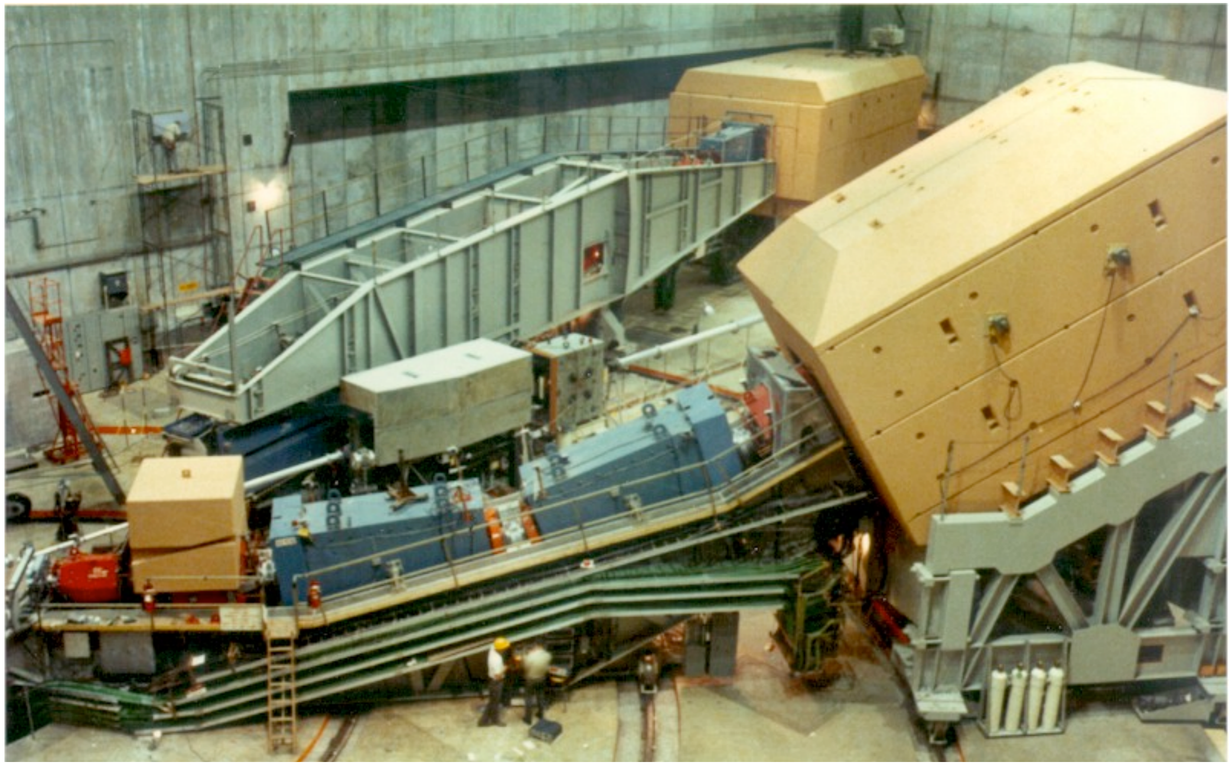
Discovery of the Quark Structure of Matter



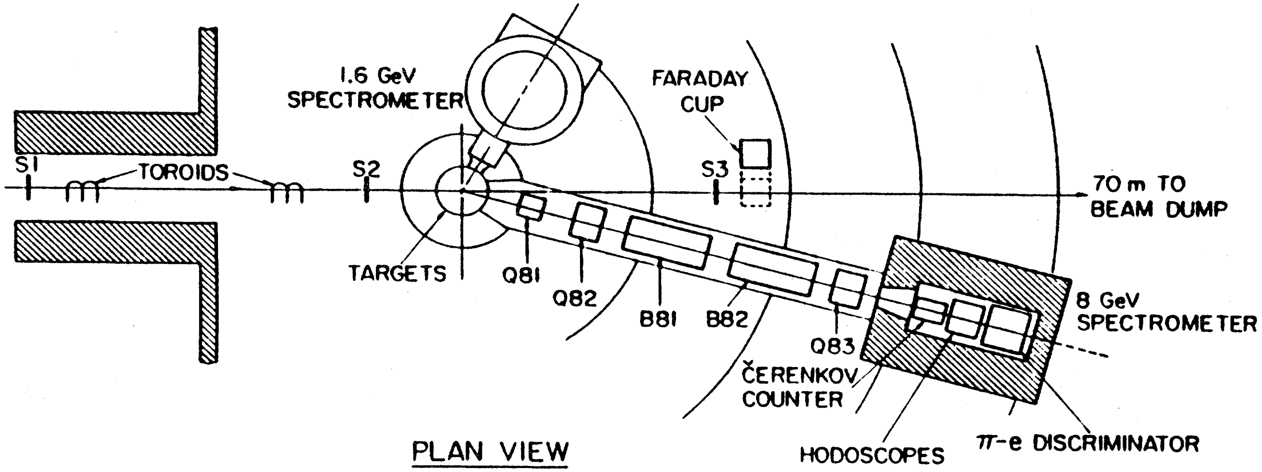
SLAC NATIONAL ACCELERATOR LABORATORY

1967

SLAC Two-Mile Linear Accelerator

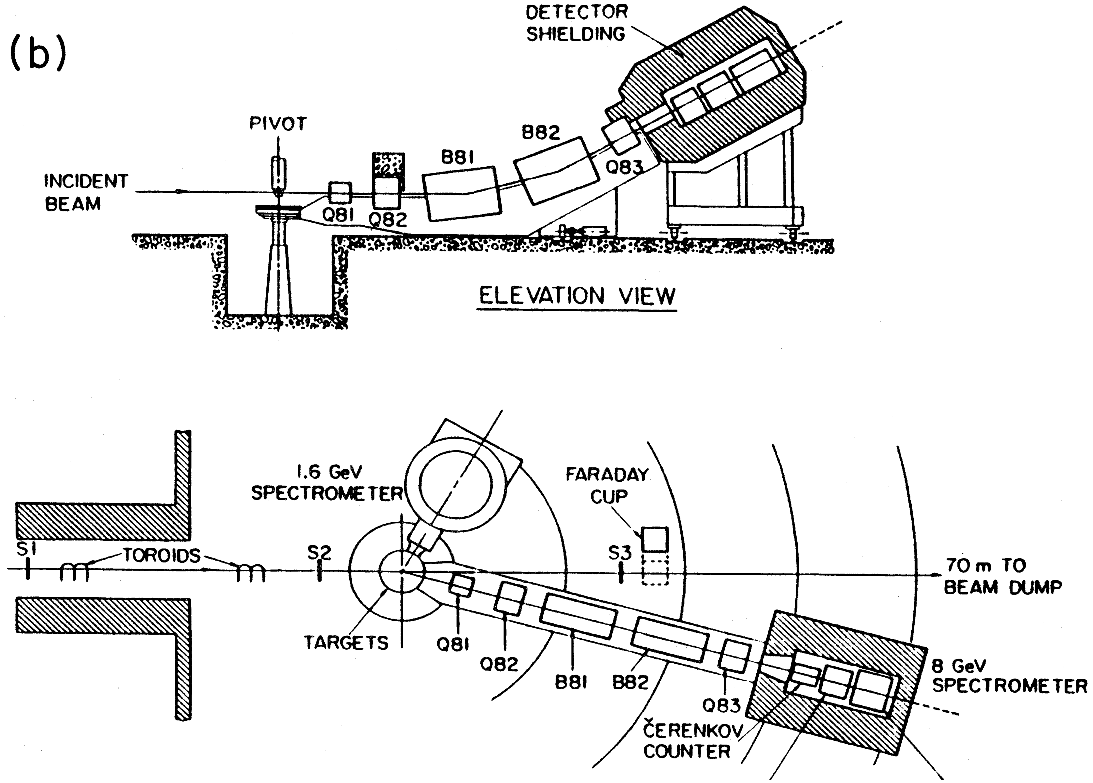


Pief

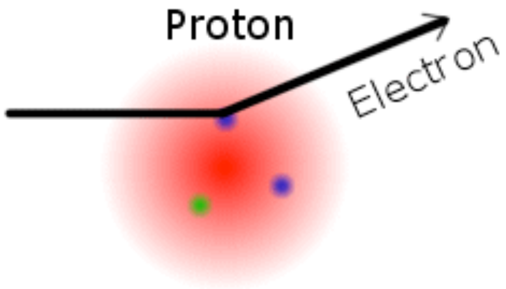


1967 SLAC Experiment:
Scatter 20 GeV/c Electrons on protons
in a Hydrogen Target
Discovery of the Quark Structure of Matter

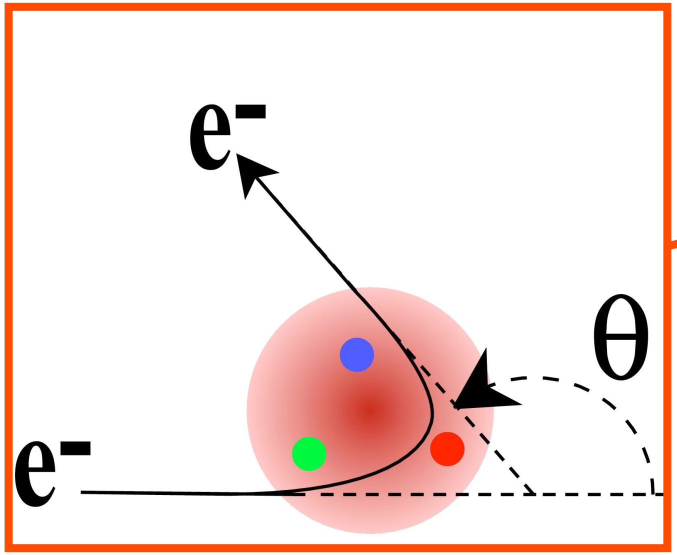
$$ep \longrightarrow e'X$$



Deep inelastic scattering: Experiments on the proton and the observation of scaling*

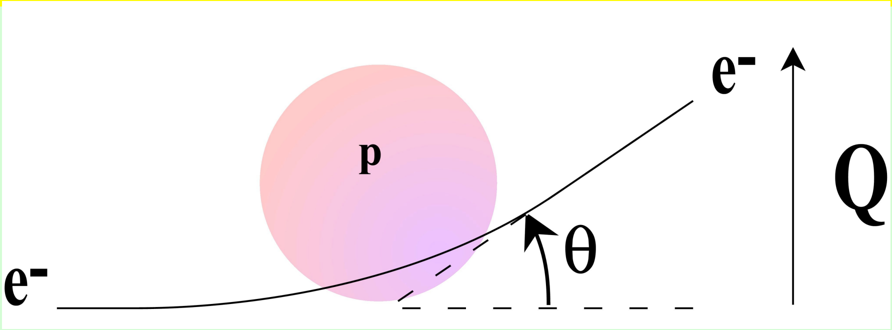


Discovery of quarks!



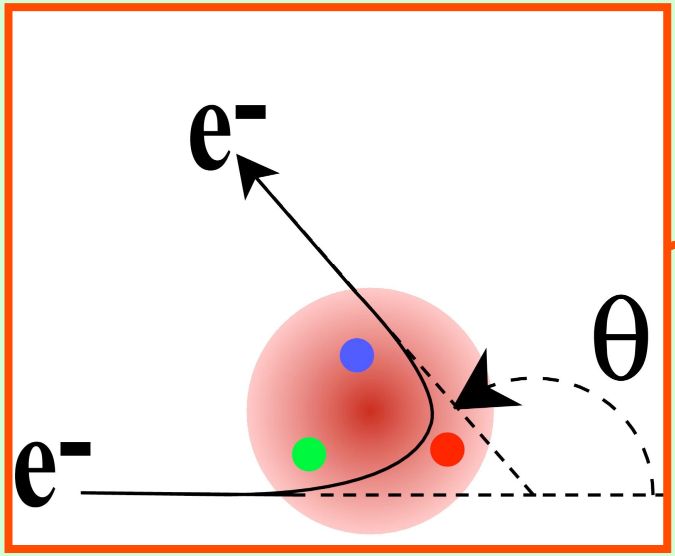
Friedman, Kendall, Taylor: Nobel Prize

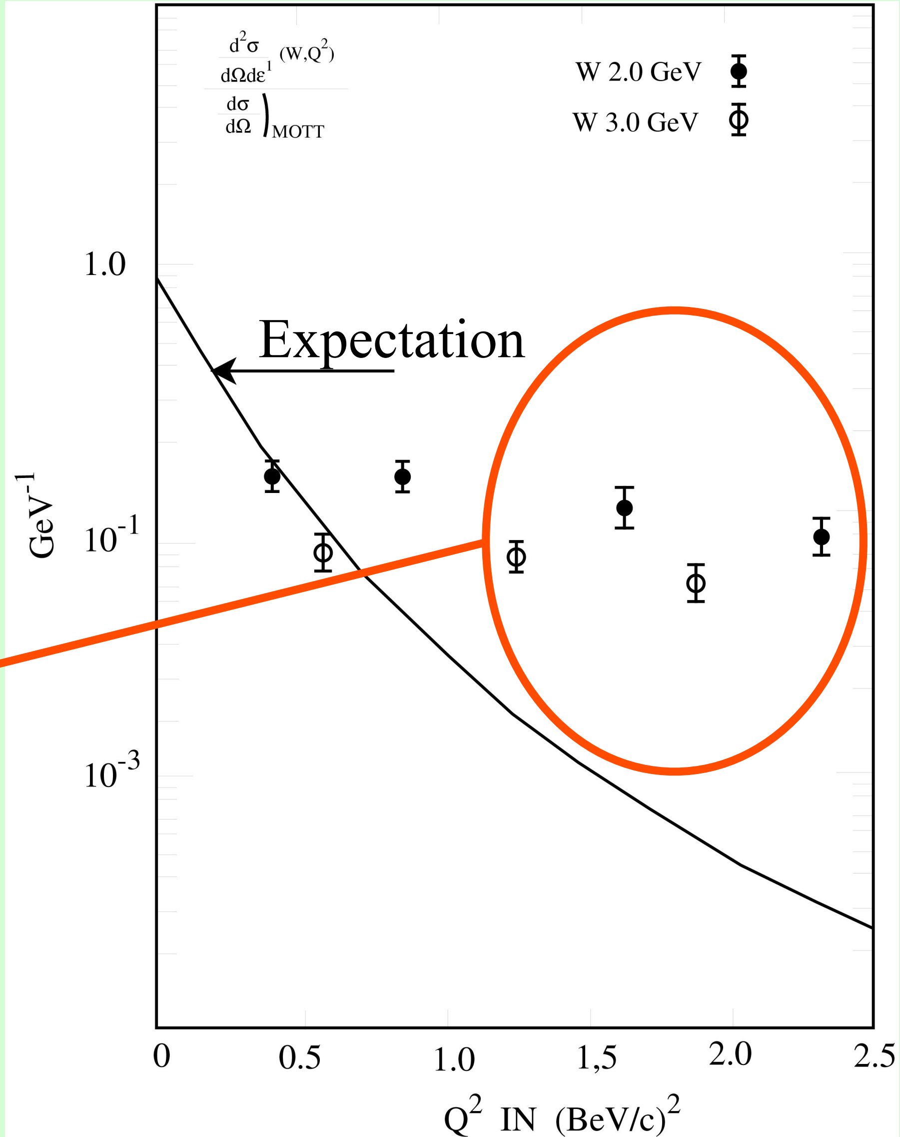
Deep inelastic electron-proton scattering



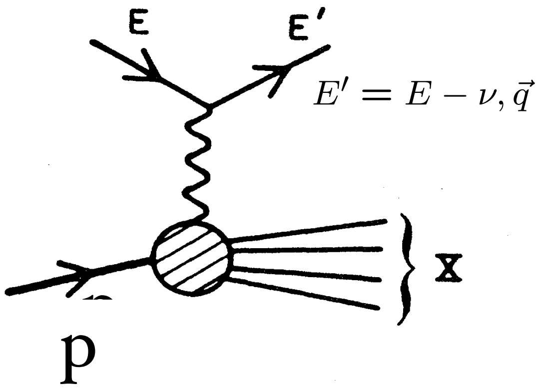
- Rutherford scattering using *very* high-energy electrons striking protons

Discovery of quarks!

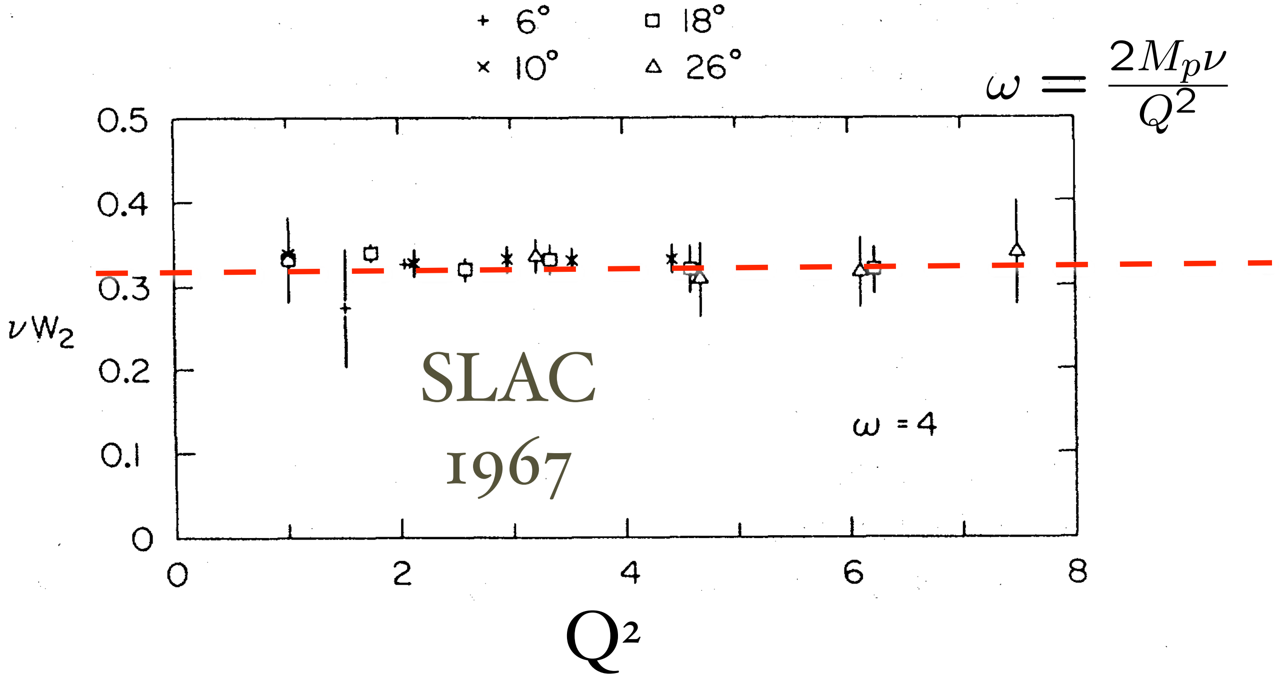




$$ep \rightarrow e'X$$



$$Q^2 = \vec{q}^2 - \nu^2$$



$$\omega = \frac{2M_p\nu}{Q^2}$$

No intrinsic length scale !

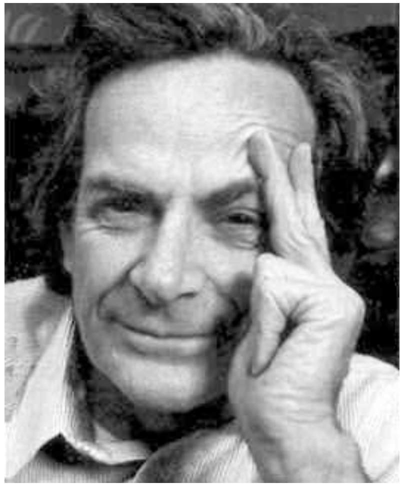
Measure rate as a function of energy loss ν and momentum transfer Q

Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!

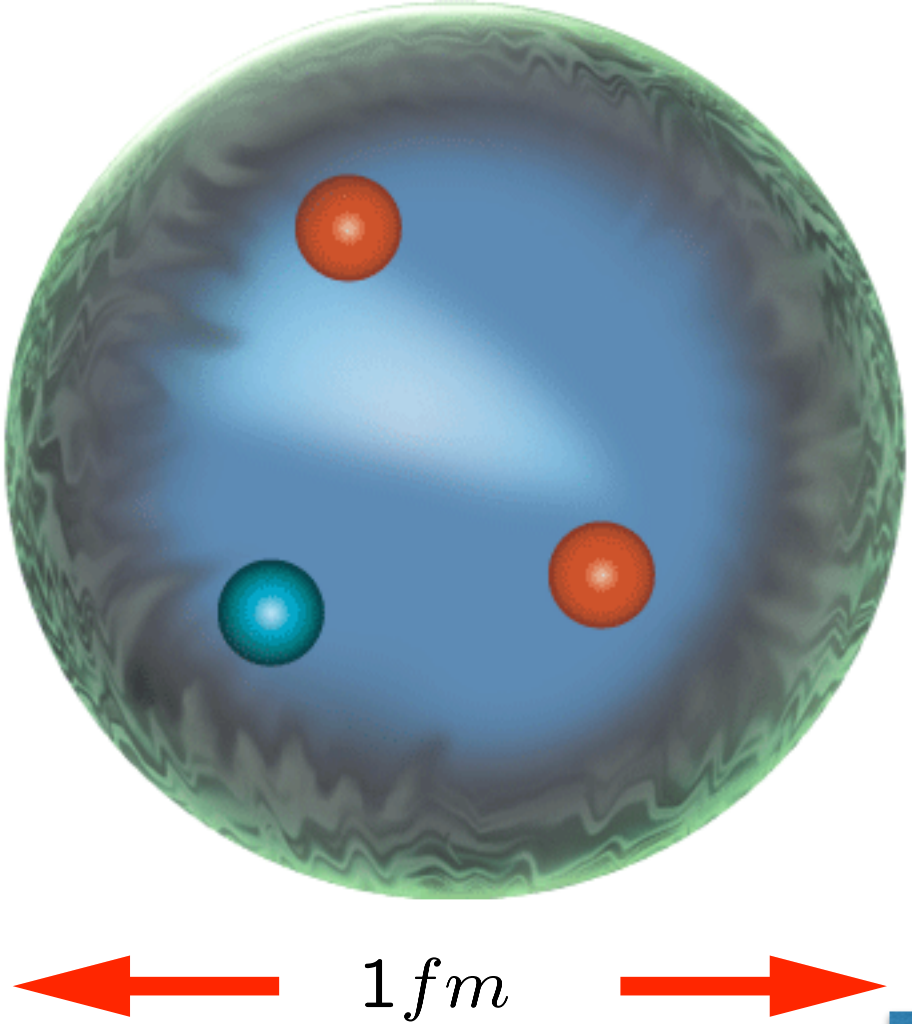
Quarks in the Proton



Feynman & Bjorken: "Parton" model

Bj: Wolf Prize, EPS Award

p = (u u d)



$1\,fm$

$10^{-15}m = 10^{-13}cm$

Zweig: "Aces, Deuces, Treys"

Gell Mann: "Three Quarks for Mr. Mark"

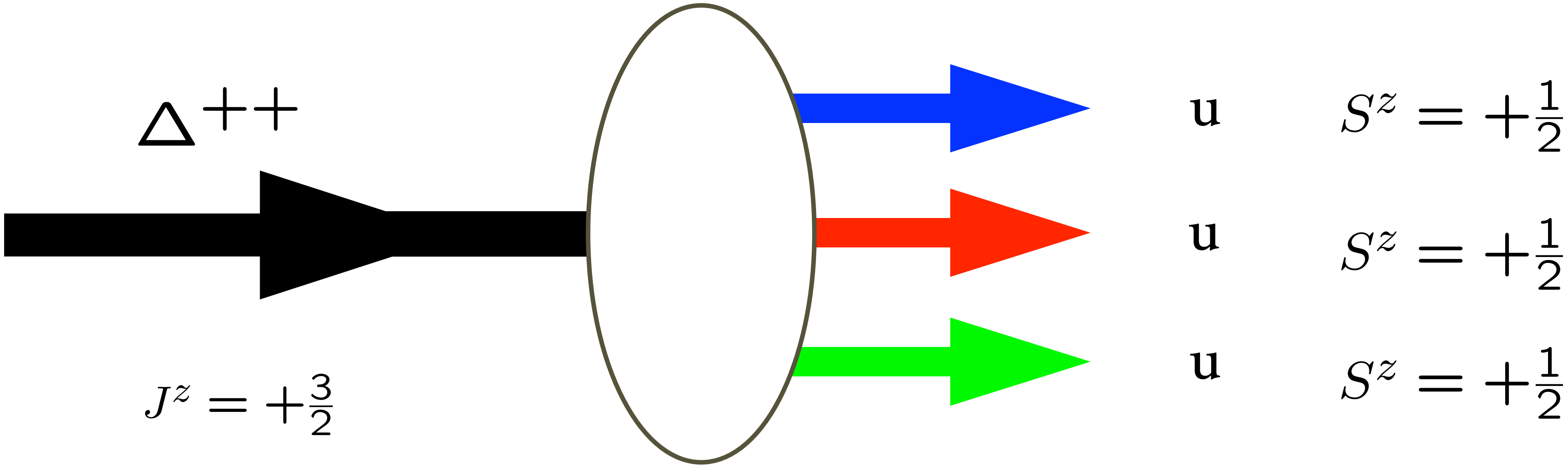
Jefferson Lab
Thomas Jefferson National Accelerator Facility

Why are there three colors of quarks?

Greenberg

Pauli Exclusion Principle!

spin-half quarks cannot be in same quantum state !



Δ^{++}

$J^z = +\frac{3}{2}$

u $S^z = +\frac{1}{2}$

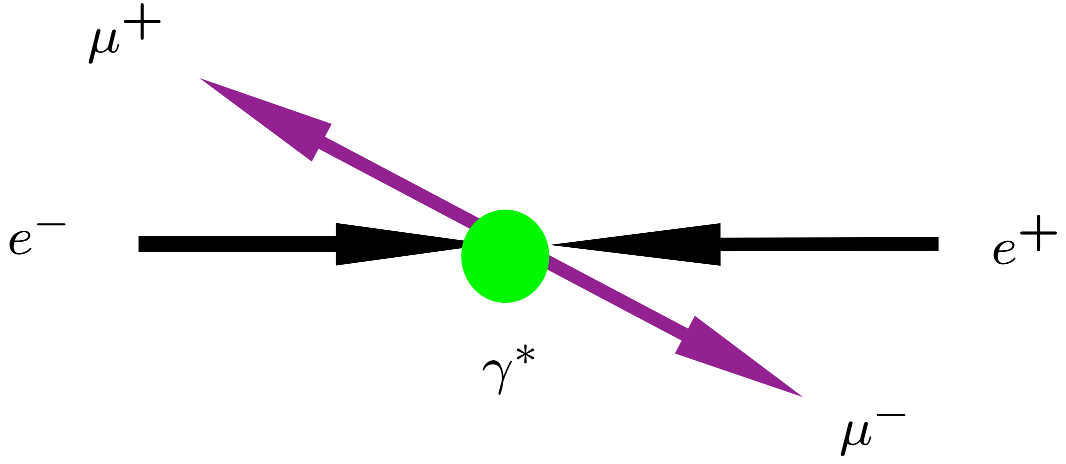
u $S^z = +\frac{1}{2}$

u $S^z = +\frac{1}{2}$

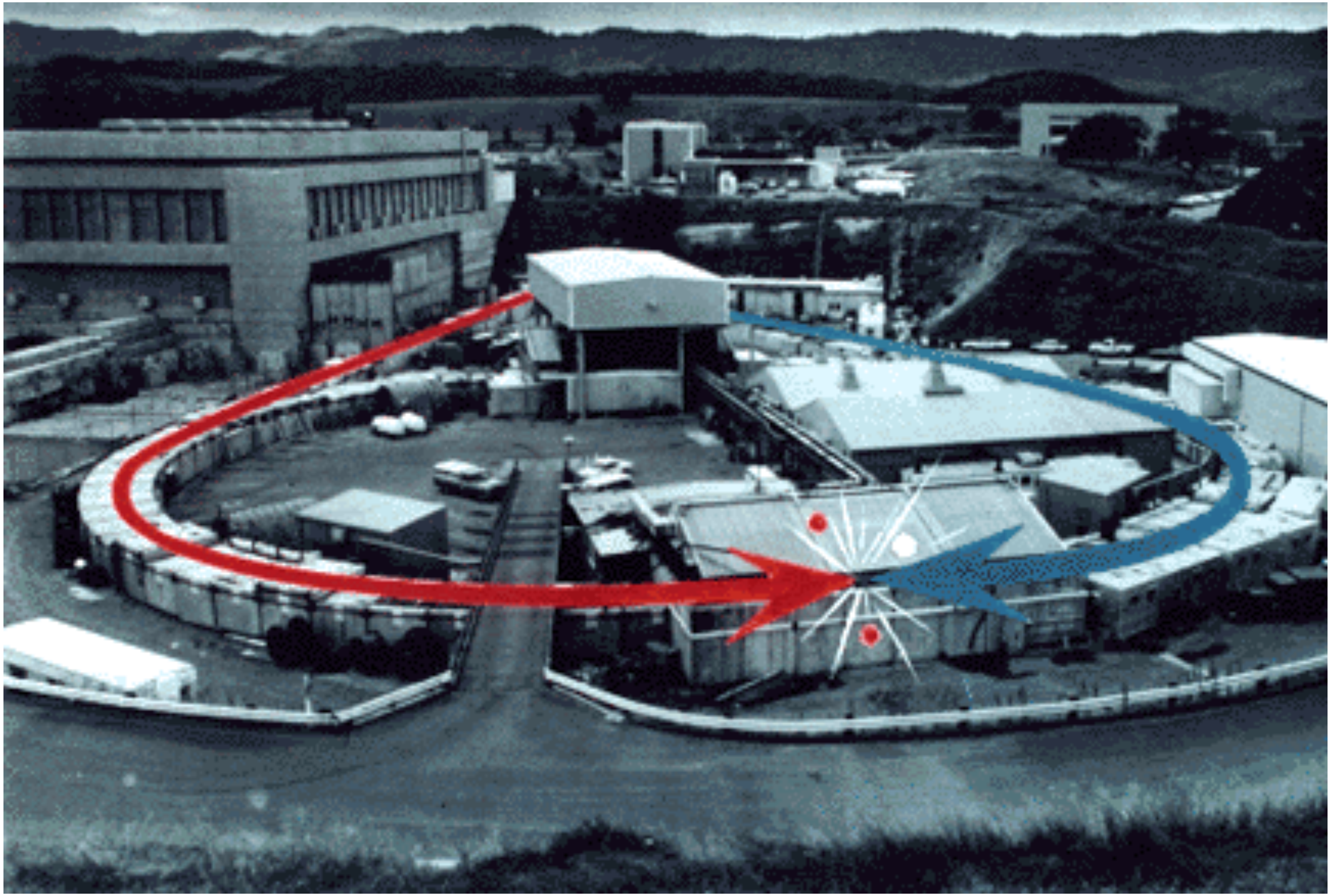
Three Colors (Parastatistics) Solves Paradox

3 Colors Combine : WHITE $SU(N_C), N_C = 3$

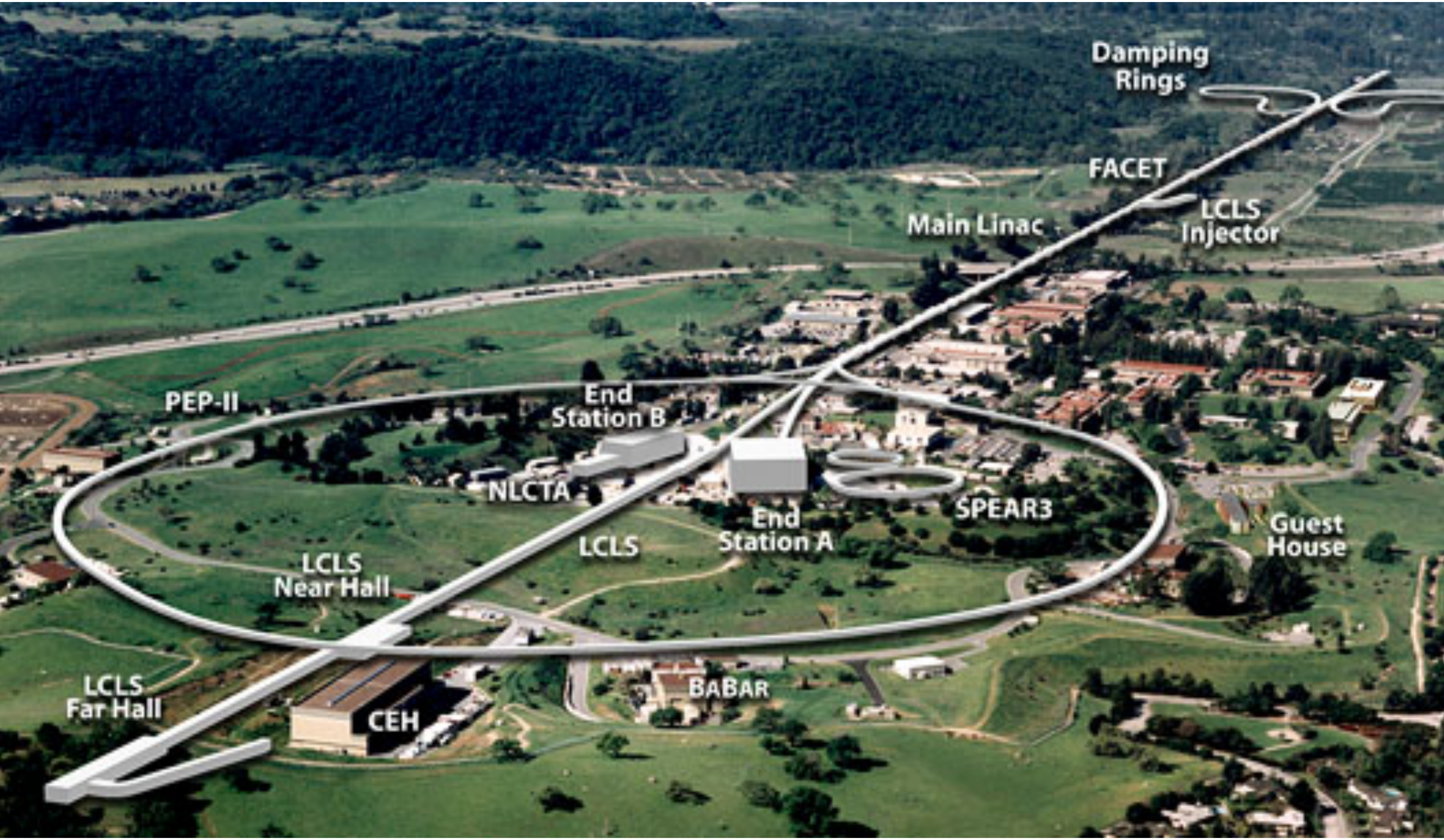
Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to \mu^+\mu^-$$

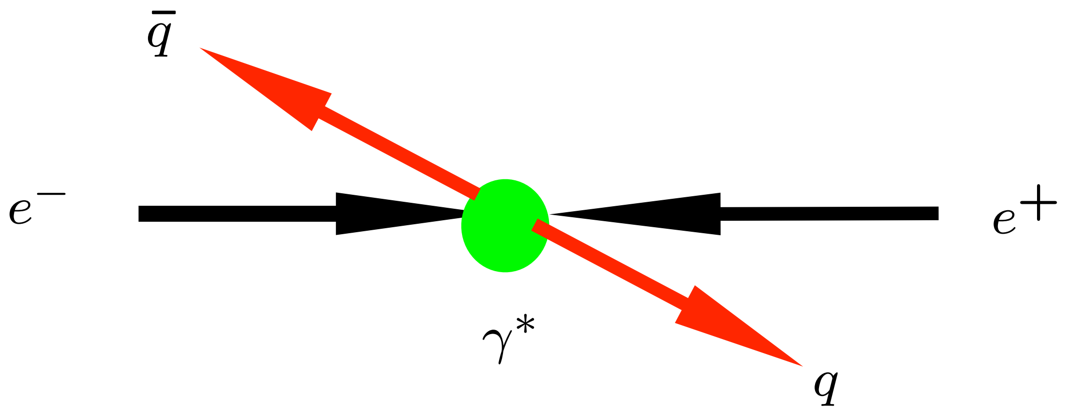


SPEAR (electron-positron collider):
discovery of the ψ ($c\ \bar{c}$), $D(c,\bar{d})$, and τ lepton



SLAC Evolution: ***SPEAR, PEP-II/BaBar, SLC/SLD, FACET, LCLS, LCLS II…***

Electron-Positron Annihilation

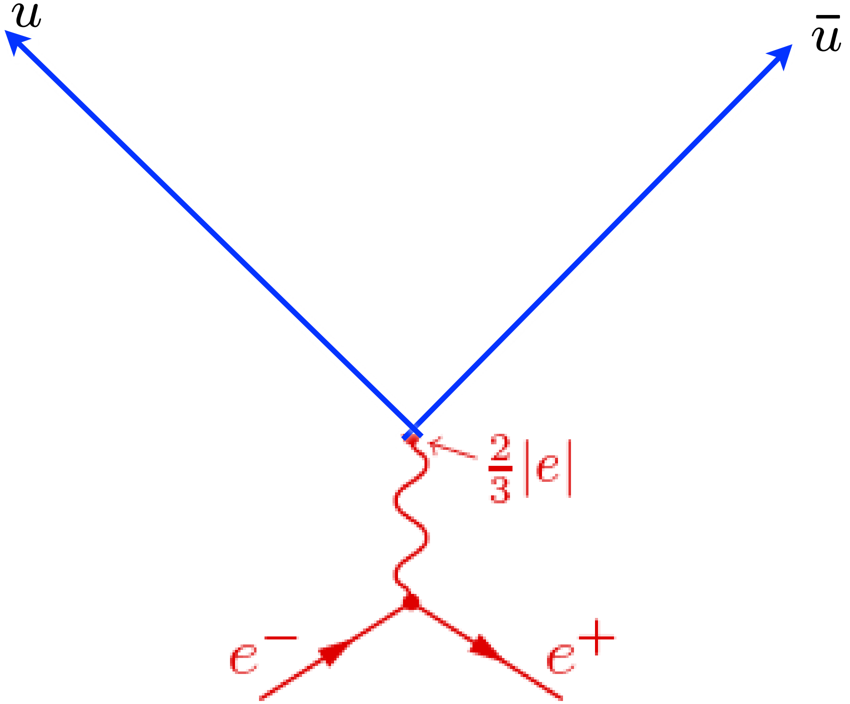


$$e^+e^- \to \gamma^* \to q\bar{q}$$

Ratio to muon pairs proportional to quark charge squared and the number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

How to Count Quarks



Color-triplet quark representation

For $10\ \text{GeV} < E_{\text{cm}} < 40\ \text{GeV}$,

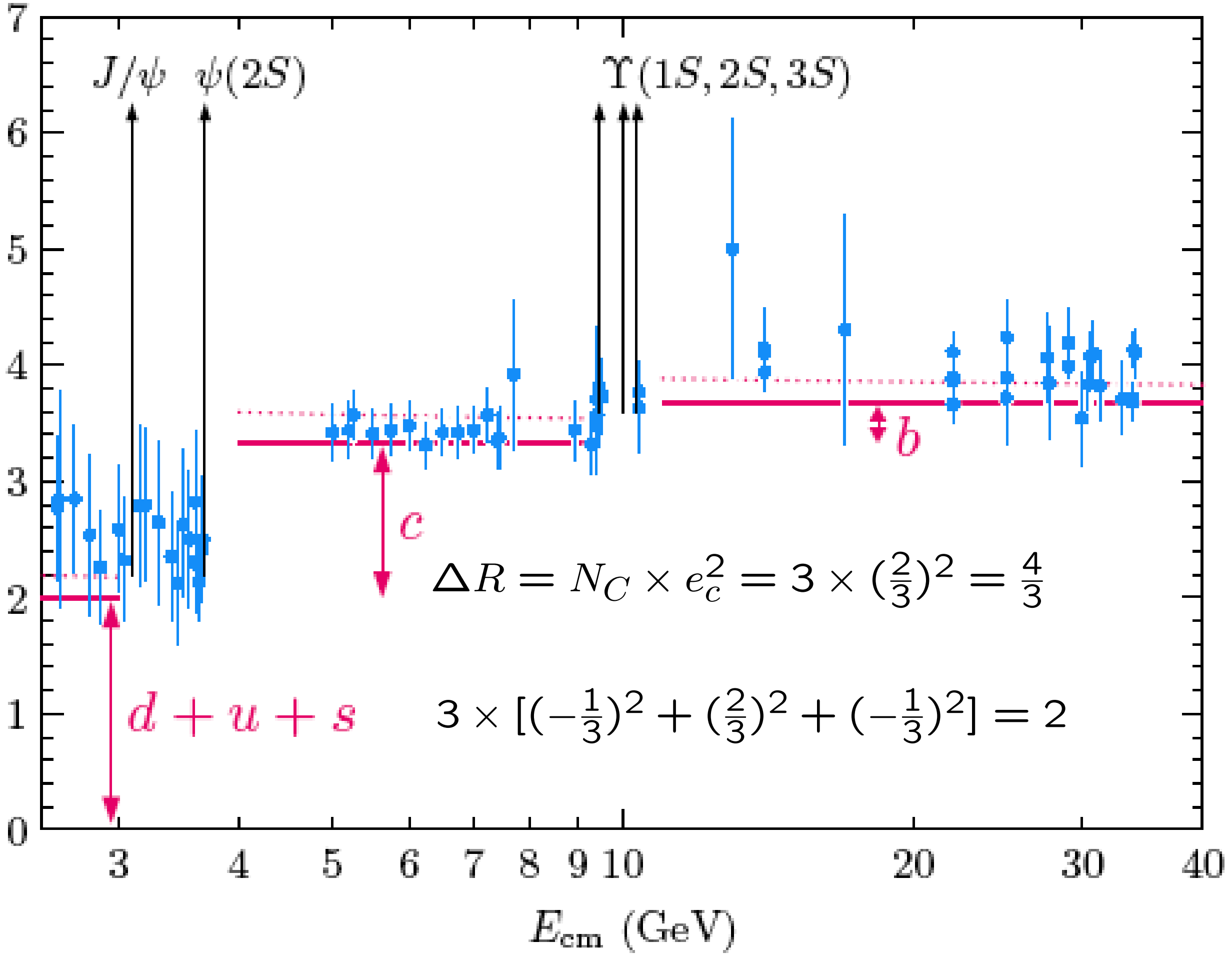
$$\frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = \underset{\substack{\uparrow \\ \text{colors}}}{3} \times \left[\underset{\substack{\uparrow \\ d}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ u}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ s}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ c}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ b}}{\left(\frac{1}{3}\right)^2} \right]$$

$J/\psi = (c\bar{c})_{1S}$

How to Count Quarks

$\Upsilon = (b\bar{b})_{1S}$

$$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$$



$$\Delta R = N_C \times e_c^2 = 3 \times (\tfrac{2}{3})^2 = \tfrac{4}{3}$$

$$3 \times [(-\tfrac{1}{3})^2 + (\tfrac{2}{3})^2 + (-\tfrac{1}{3})^2] = 2$$

$$3 \times (-\tfrac{1}{3})^2 = \tfrac{1}{3}$$

$$\boxed{N_C = 3}$$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

Primary Evidence for Quarks

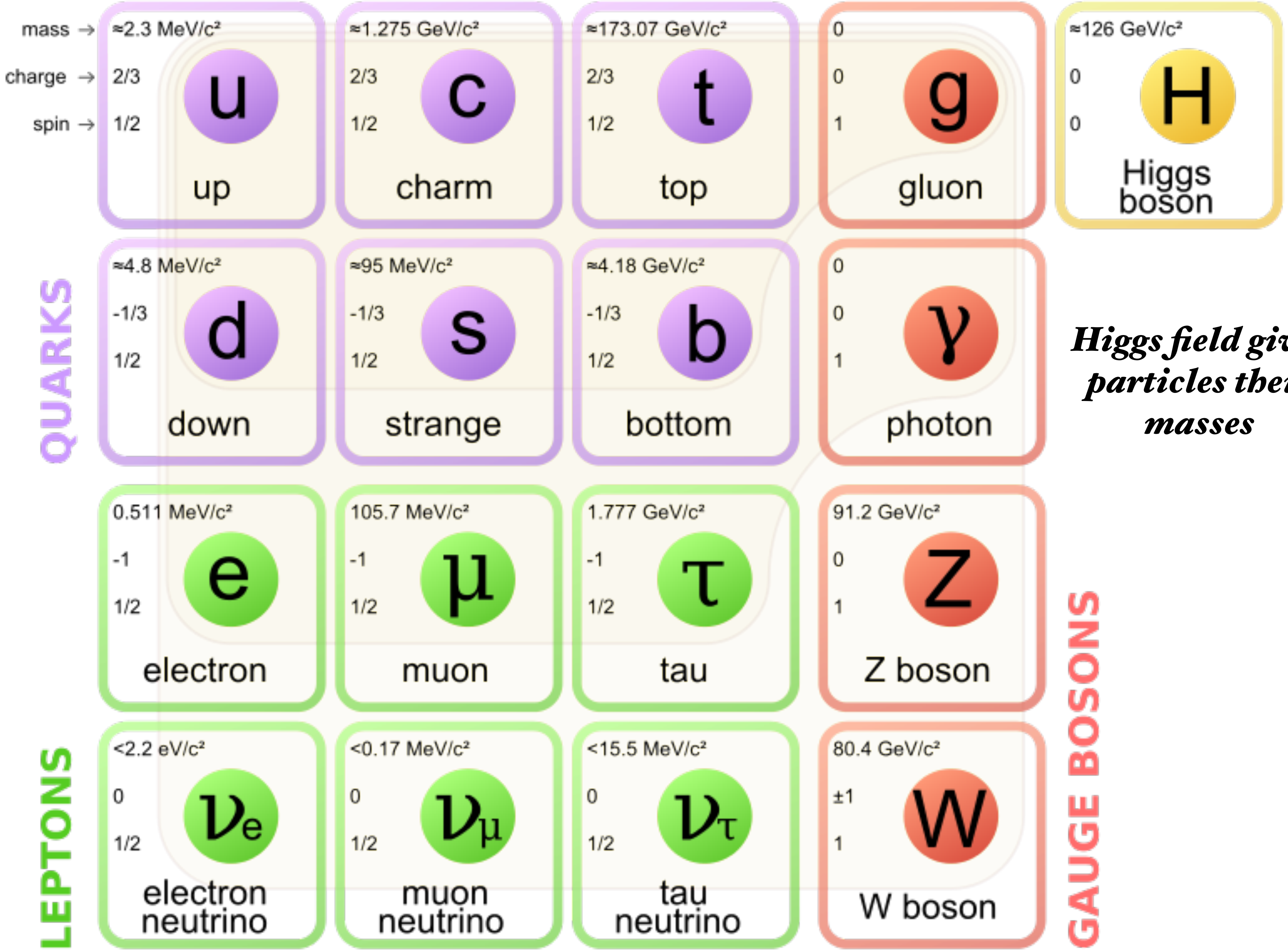
- Electron-Proton Inelastic Scattering: $ep \to e'X$
  Electron scatters on pointlike constituents with fractional charge; final-state jets

- Electron-Positron Annihilation: $e^+e^- \to X$
  Production of pointlike pairs with fractional charges and 3 colors; quark, antiquark, gluon jets

- Exclusive hard scattering reactions: $pp \to pp, \ \gamma p \to \pi^+ n, \ ep \to ep$
  probability that hadron stays intact counts number of its pointlike constituents: Quark Counting Rules

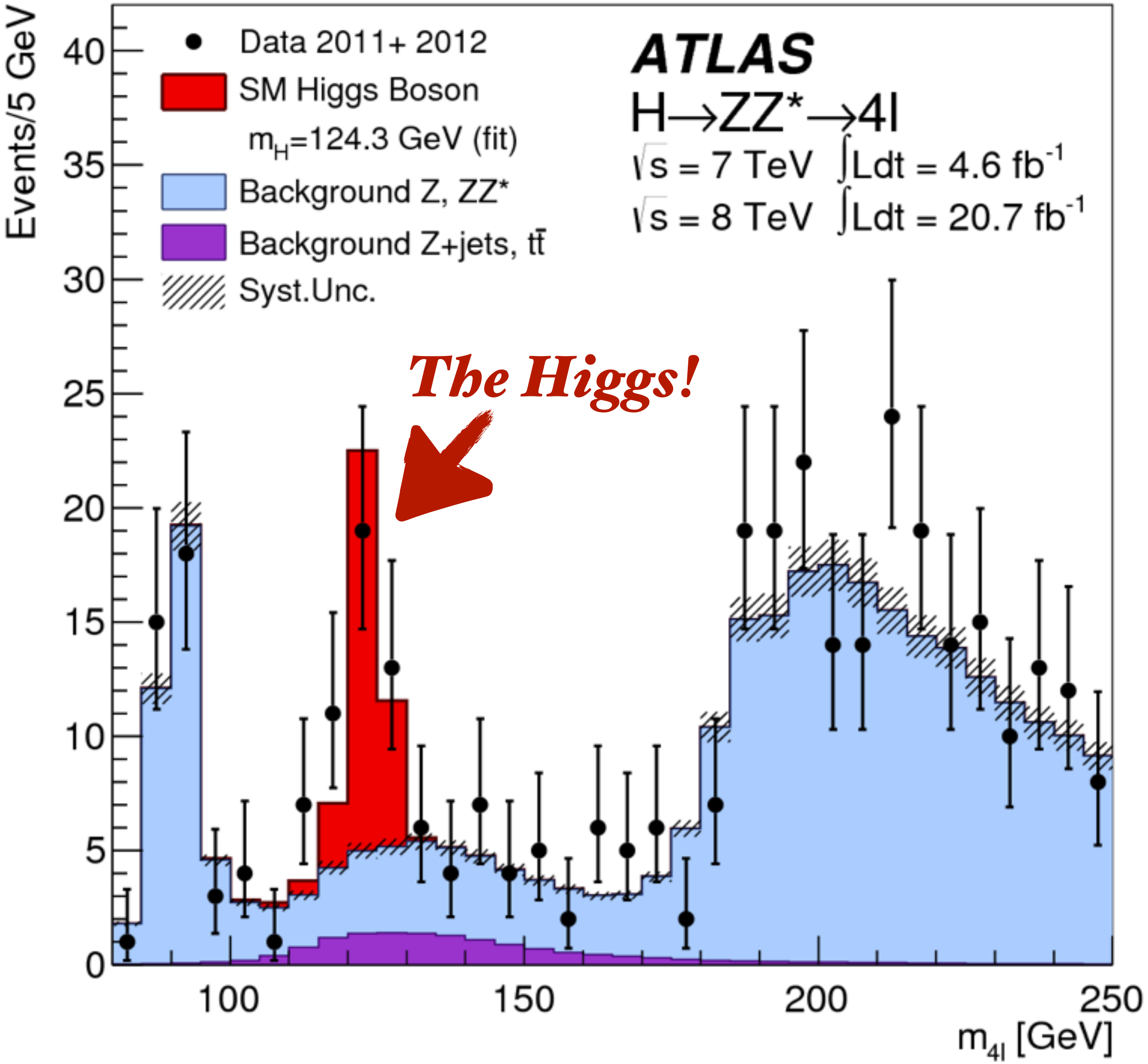
Quark interchange describes angular distribution

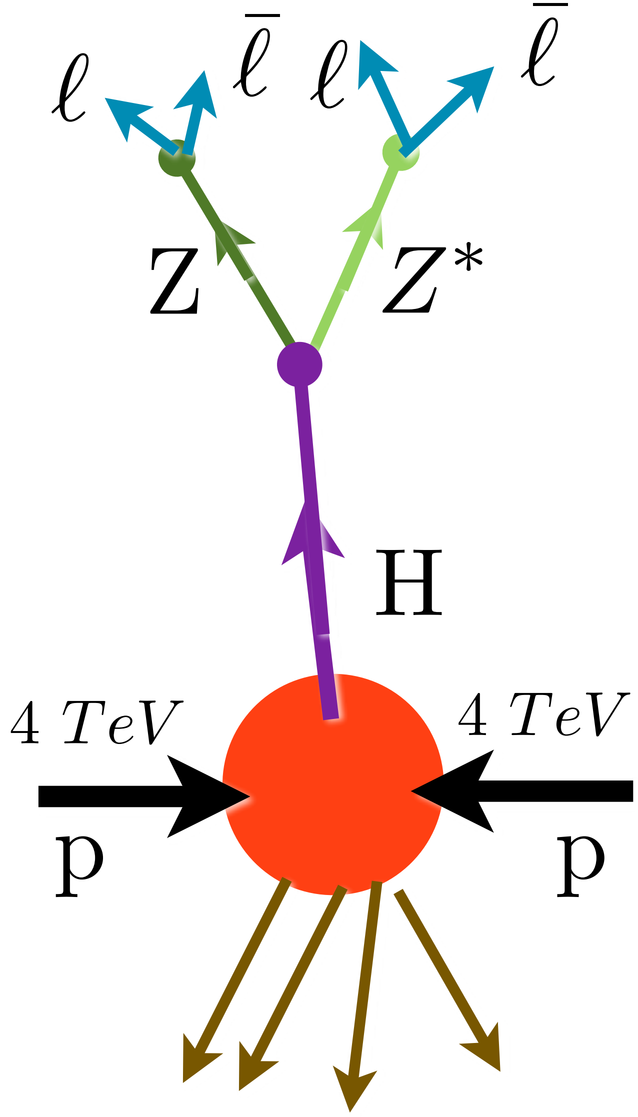
Fundamental Constituents underlying atoms, nuclei, and hadrons



mass → charge → spin →	≈2.3 MeV/c^2 2/3 1/2 u up	≈1.275 GeV/c^2 2/3 1/2 c charm	≈173.07 GeV/c^2 2/3 1/2 t top	0 0 1 g gluon	≈126 GeV/c^2 0 0 H Higgs boson
	≈4.8 MeV/c^2 -1/3 1/2 d down	≈95 MeV/c^2 -1/3 1/2 s strange	≈4.18 GeV/c^2 -1/3 1/2 b bottom	0 0 1 γ photon	
	0.511 MeV/c^2 -1 1/2 e electron	105.7 MeV/c^2 -1 1/2 μ muon	1.777 GeV/c^2 -1 1/2 τ tau	91.2 GeV/c^2 0 1 Z Z boson	
	<2.2 eV/c^2 0 1/2 ν_e electron neutrino	<0.17 MeV/c^2 0 1/2 ν_μ muon neutrino	<15.5 MeV/c^2 0 1/2 ν_τ tau neutrino	80.4 GeV/c^2 ±1 1 W W boson	

Higgs field gives particles their masses

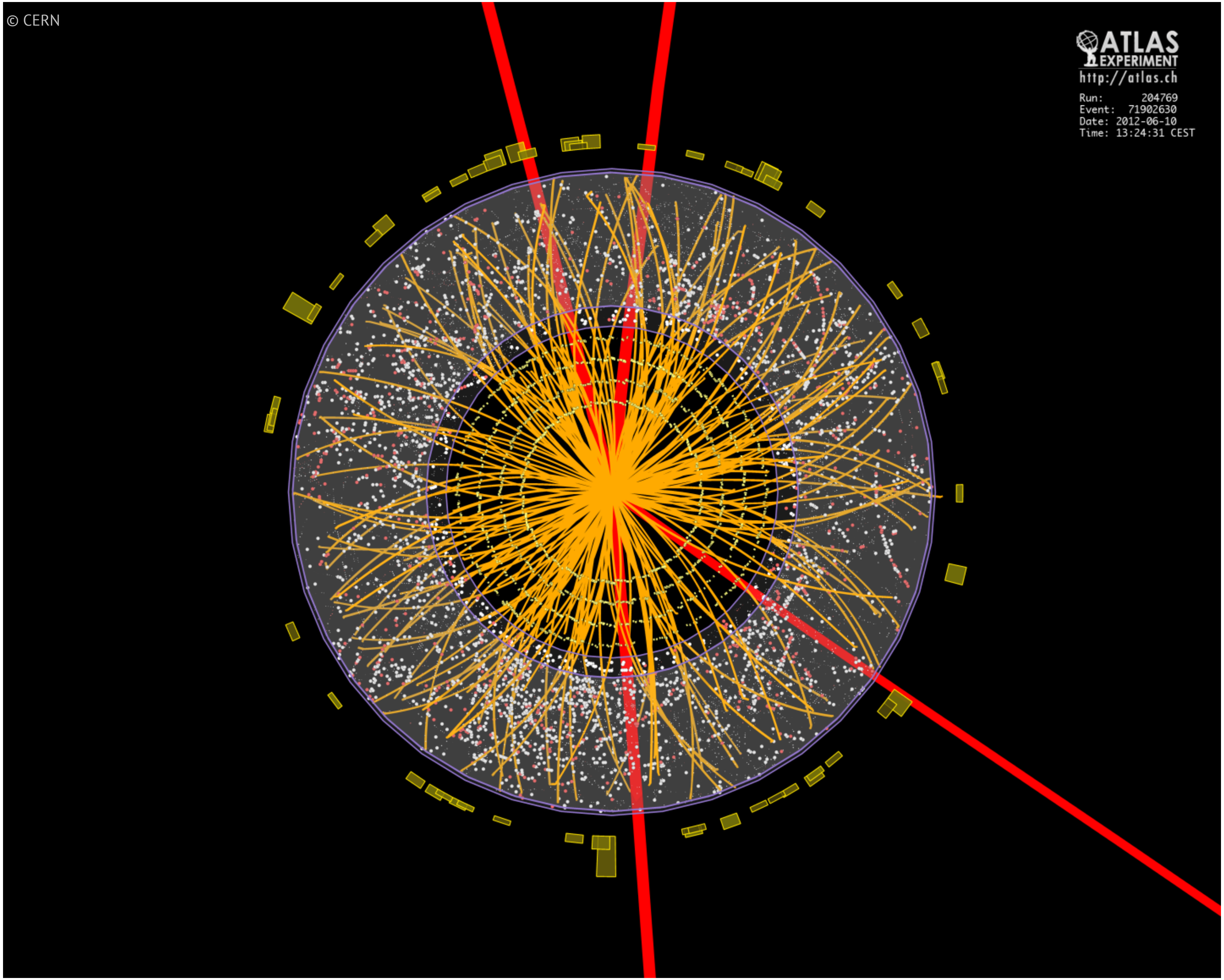




$1\ event\ in\ 10^{10}$

Novel World of Hadron Physics

Stan Brodsky



ATLAS $pp \longrightarrow H \longrightarrow \mu^+\mu^-\mu^+\mu^- X$

QED Lagrangian

$$\boxed{\mathcal{L}_{QED} = -\frac{1}{4} Tr(F^{\mu\nu} F_{\mu\nu}) + \sum_{\ell=1}^{n_\ell} i \bar{\Psi}_\ell D_\mu \gamma^\mu \Psi_\ell + \sum_{\ell=1}^{n_\ell} m_\ell \bar{\Psi}_\ell \Psi_\ell}$$

$$iD^\mu = i\partial^\mu - eA^\mu \qquad F^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu$$

Yang Mills Gauge Principle:
Phase Invariance at Every Point
of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Landau Pole

QCD Lagrangian

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

QED: Underlies Atomic Physics, Molecular Physics, Chemistry, Electromagnetic Interactions ...

QCD: Underlies Hadron Physics, Nuclear Physics, Strong Interactions, Jets

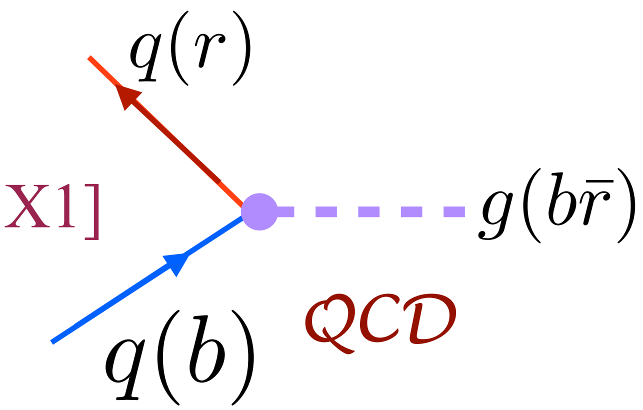
Theoretical Tools

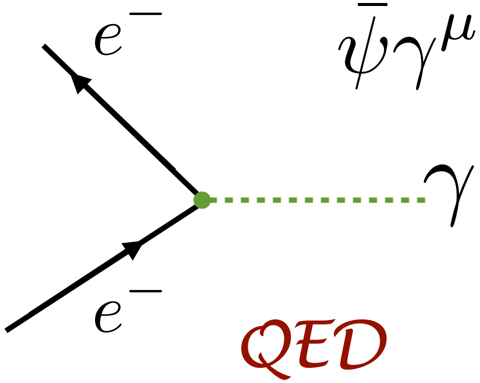
- **Feynman diagrams and perturbation theory**
- **Bethe Salpeter Equation, Dyson-Schwinger Equations**
- **Lattice Gauge Theory,**
- **Discretized Light-Front Quantization**
- **AdS/QCD !**

Fundamental Couplings of QCD and QED

$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

[1X3] [3X3] [3X1]





$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

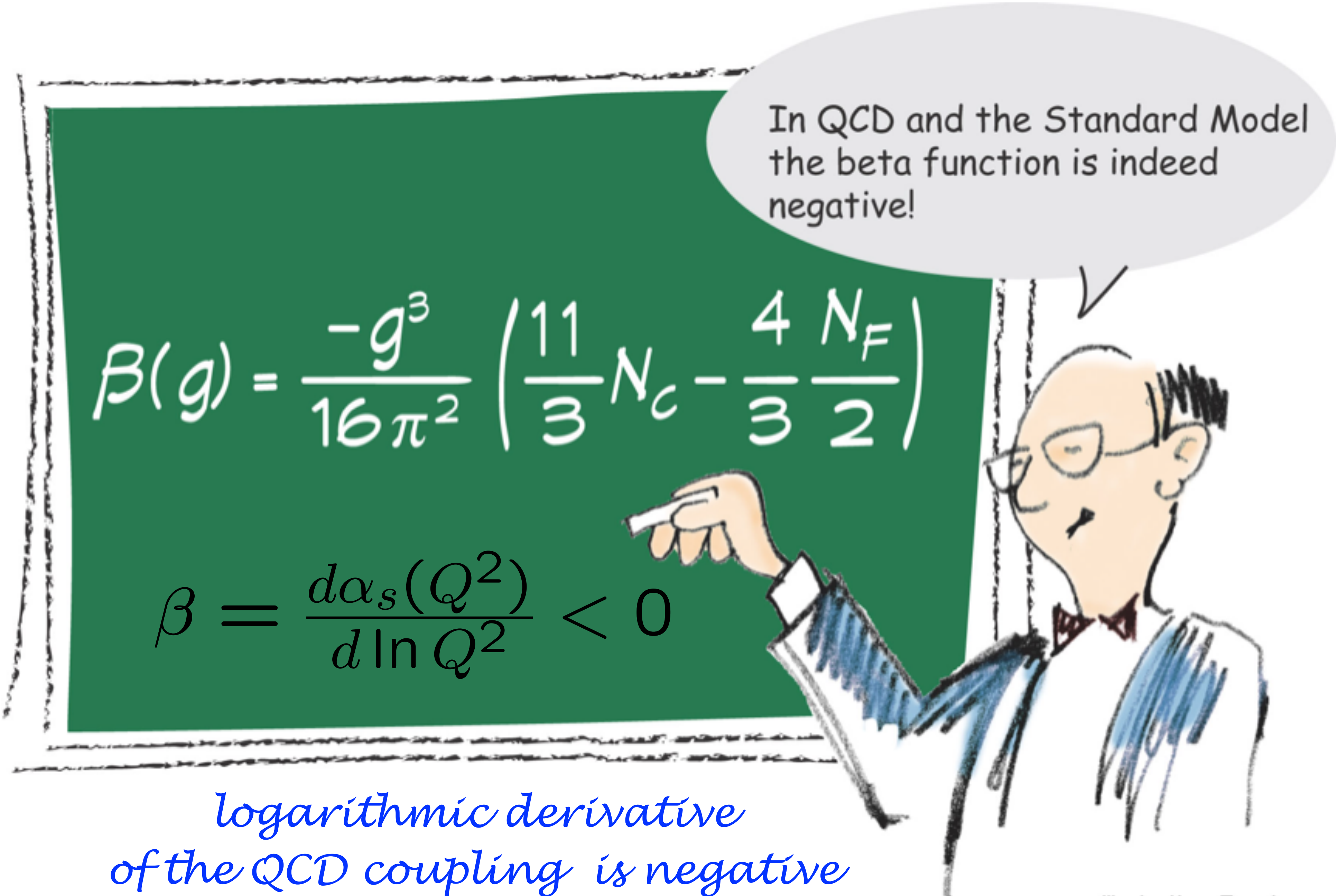
$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_{\mu}\gamma^{\mu}\Psi_f + \sum_{f=1}^{n_f} m_f\bar{\Psi}_f\Psi_f$$

$$G^{\mu\nu} = \partial^{\mu}A^{\mu} - \partial^{\nu}A^{\mu} - g[A^{\mu}, A^{\nu}]$$

Gluon vertices

$G^{\mu\nu}G_{\mu\nu}$

gluon self couplings



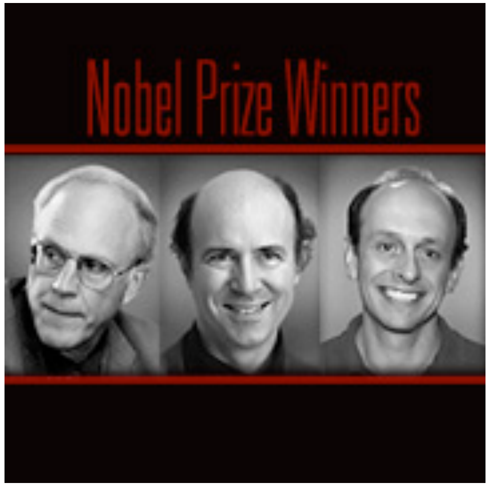
$$\beta = \frac{d\alpha_s(Q^2)}{d\ln Q^2} < 0$$

Illustration: Typoform

logarithmic derivative
of the QCD coupling is negative
Coupling becomes weaker at short
distances = high momentum transfer

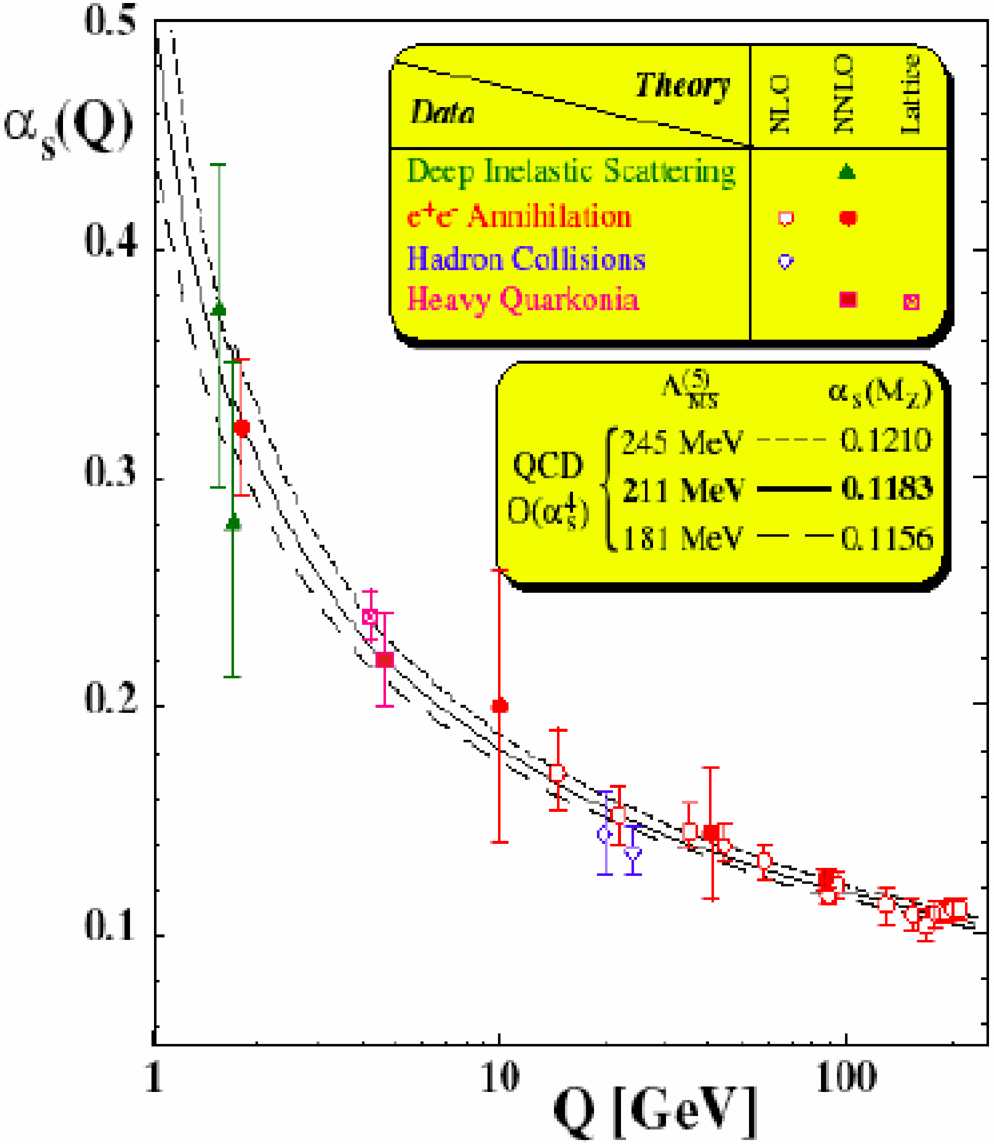
Verification of Asymptotic Freedom

$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$



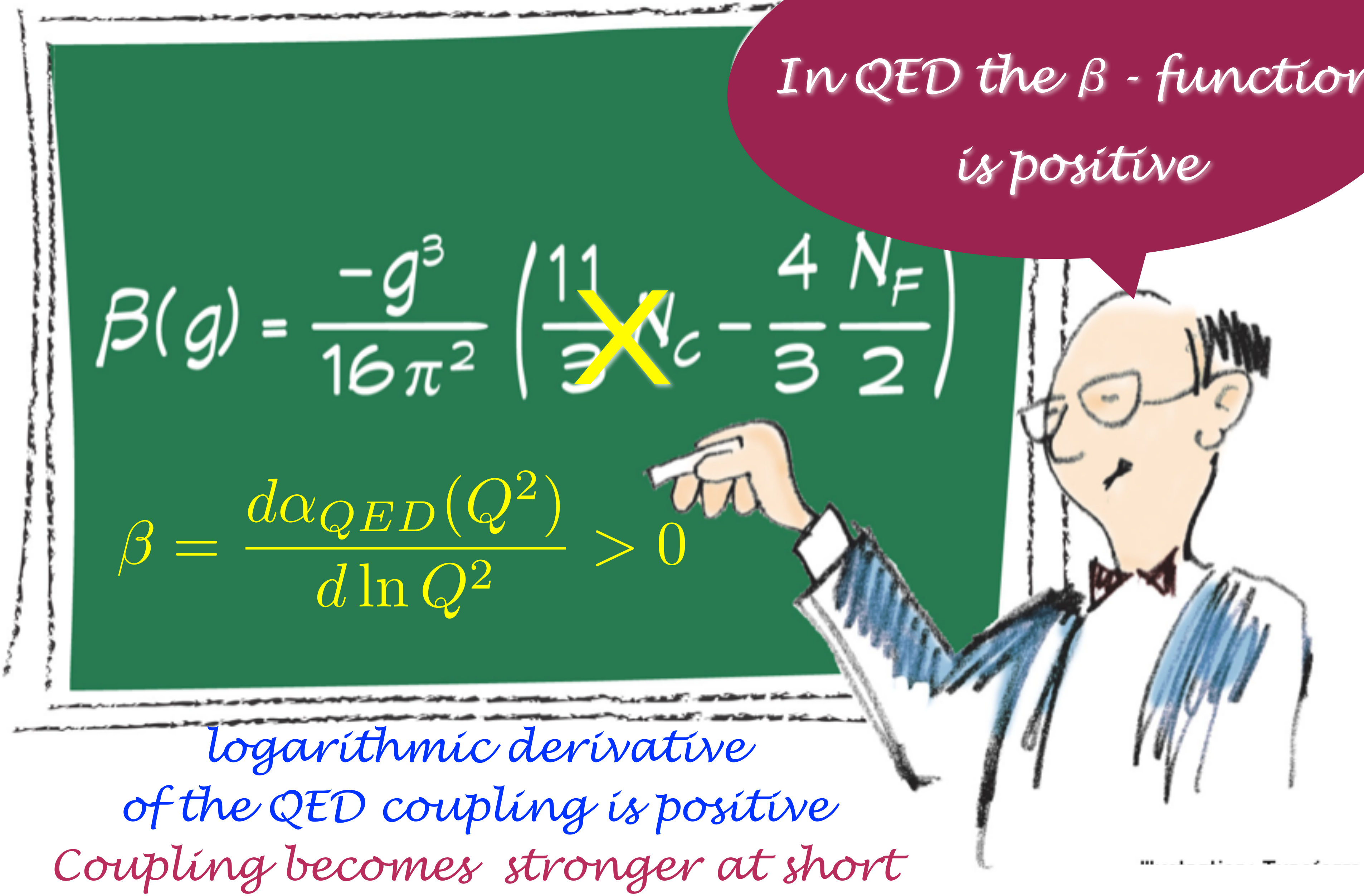
$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

proportional to $\alpha_s(Q)$



Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$

In QED the β - function is positive



$$\beta(g) = \frac{-g^3}{16\pi^2}\left(\cancel{\frac{11}{3}N_c} - \frac{4}{3}\frac{N_F}{2}\right)$$

$$\beta = \frac{d\alpha_{QED}(Q^2)}{d\ln Q^2} > 0$$

logarithmic derivative
of the QED coupling is positive

Coupling becomes stronger at short distances = high momentum transfer

Landau Pole!

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\boxed{\text{lim } N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F}$$

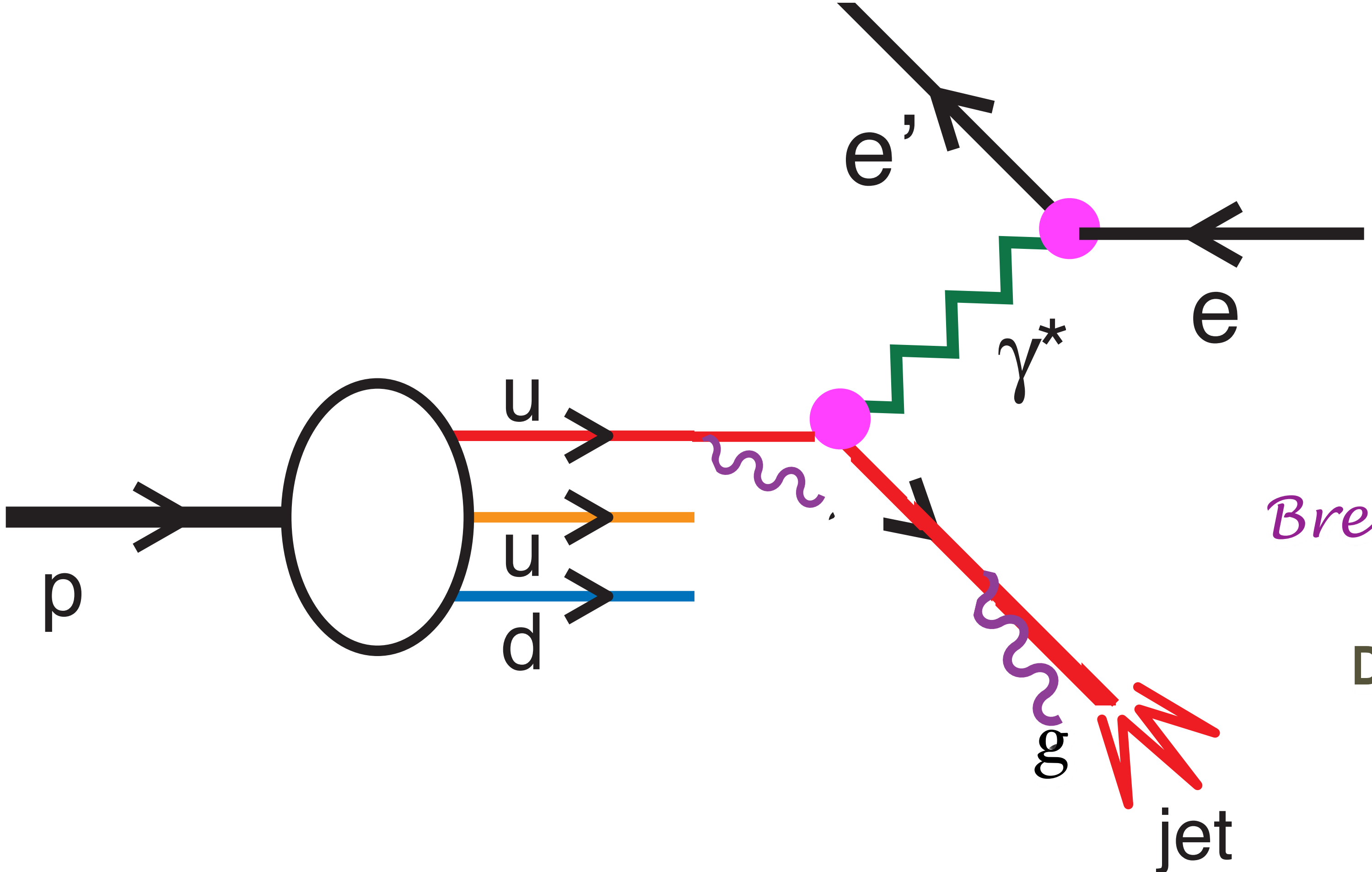
$$\boxed{\text{QCD} \to \text{Abelian Gauge Theory}}$$

Analytic Feature of SU(Nc) Gauge Theory

QCD ⟶ QED

All analyses for Quantum Chromodynamics must be applicable to Quantum Electrodynamics

First Evidence for Quark Structure of Matter



Gluonic Bremmstrahlung

DGLAP Evolution

Deep Inelastic Electron-Proton Scattering

But why do quarks not appear in the final state ?
Why are quarks confined within hadrons?

- **What is the origin of quark confinement?**
- **What determines the QCD mass scale?**
- **Novel hadronic states: tetraquarks!**
- **Novel QCD phenomena**
- **Supersymmetry in hadron physics**
- **Light-Front Holography**
- **New Physics Opportunities at JLab**

Each element of flash photograph illuminated along the light front

at a fixed

$$\tau = t + z/c$$

Evolve in LF time

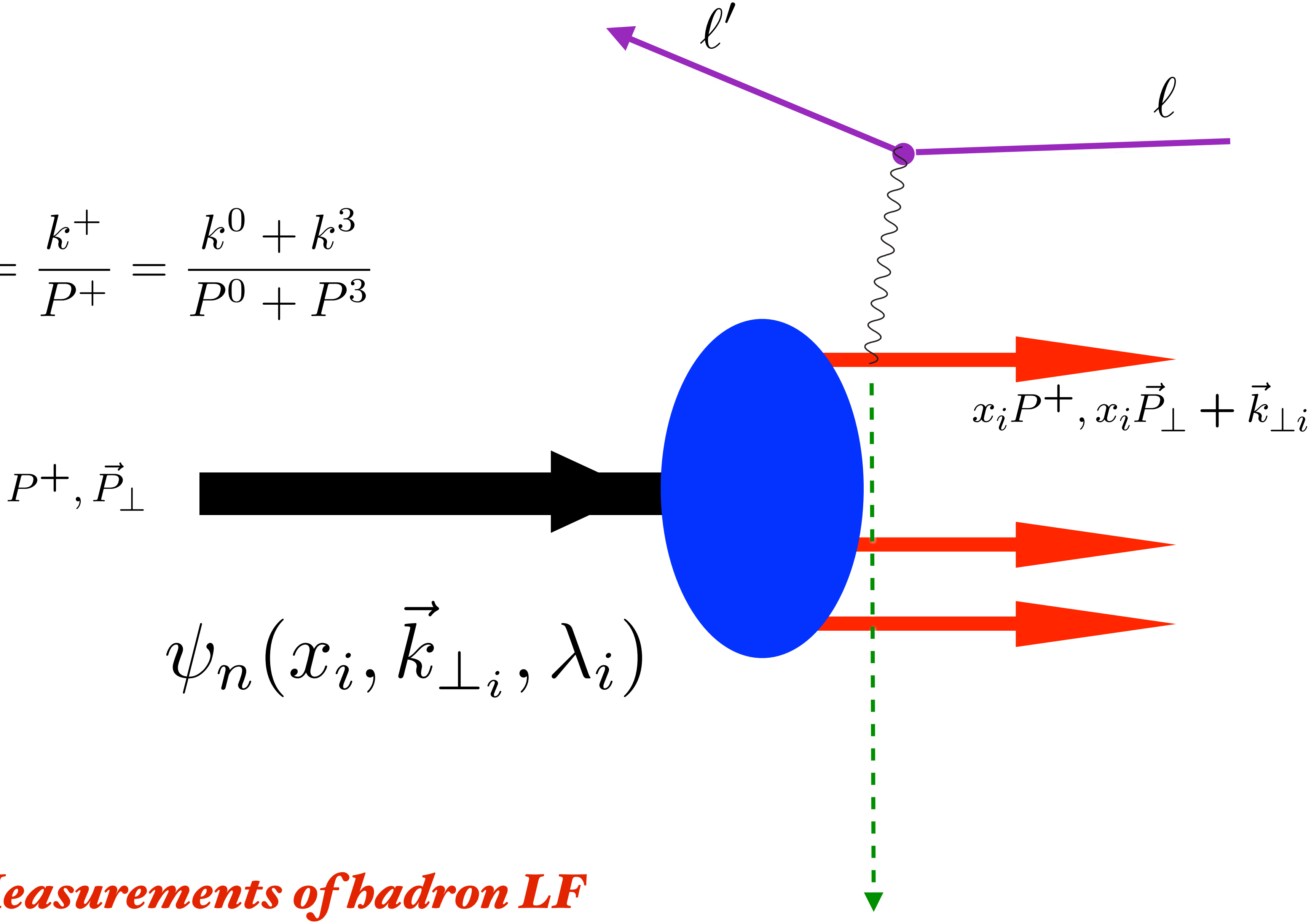
$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H^{QCD}_{LF}|\Psi_h >= \mathcal{M}^2_h|\Psi_h >$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

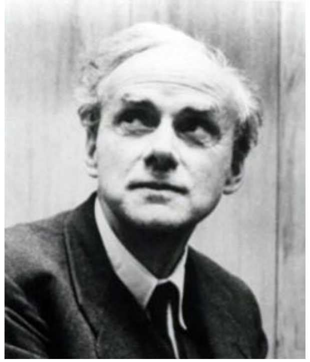
Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

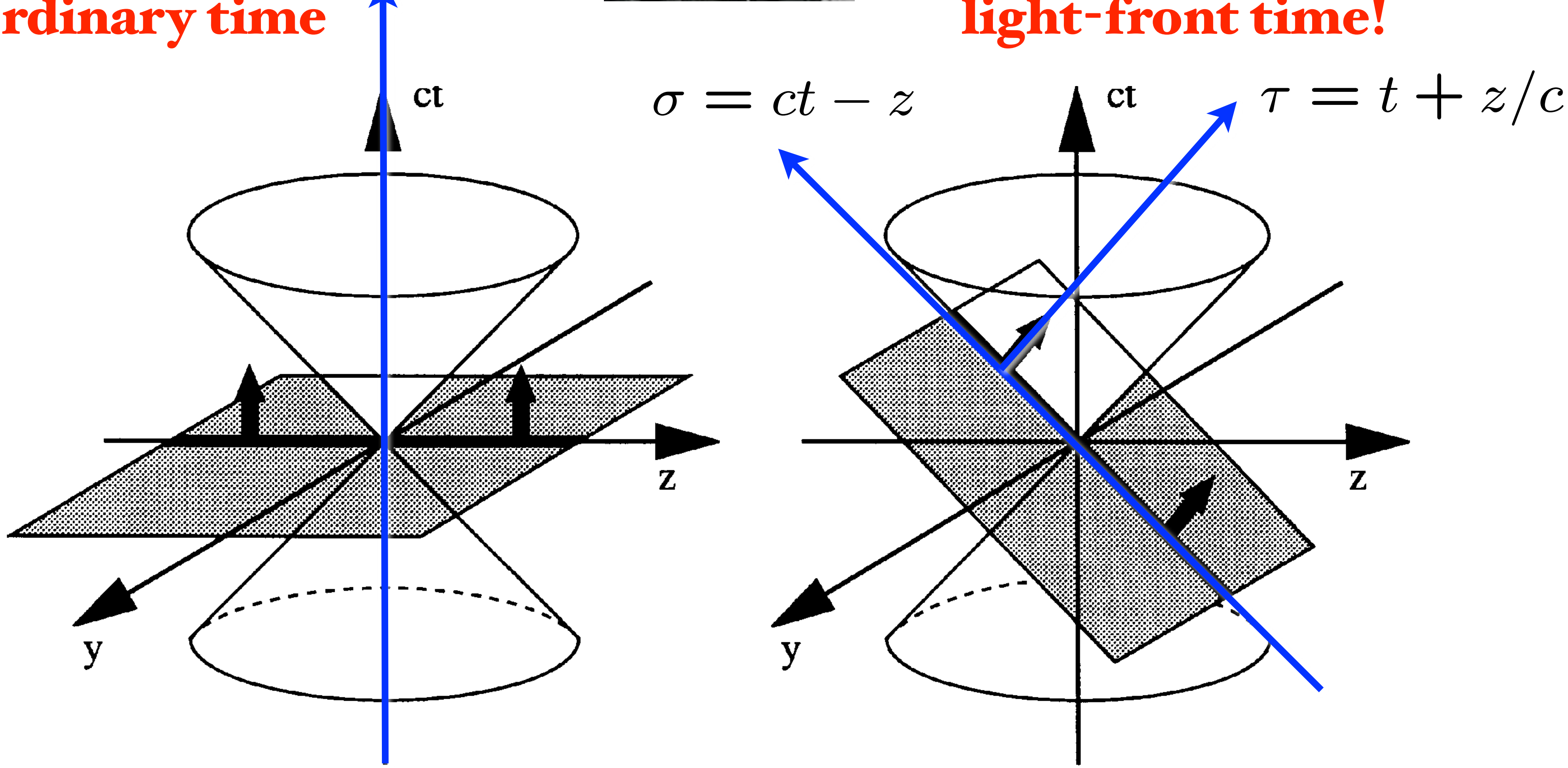
Dirac's Amazing Idea: The "Front Form"



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time

Evolve in light-front time!



$\sigma = ct - z$

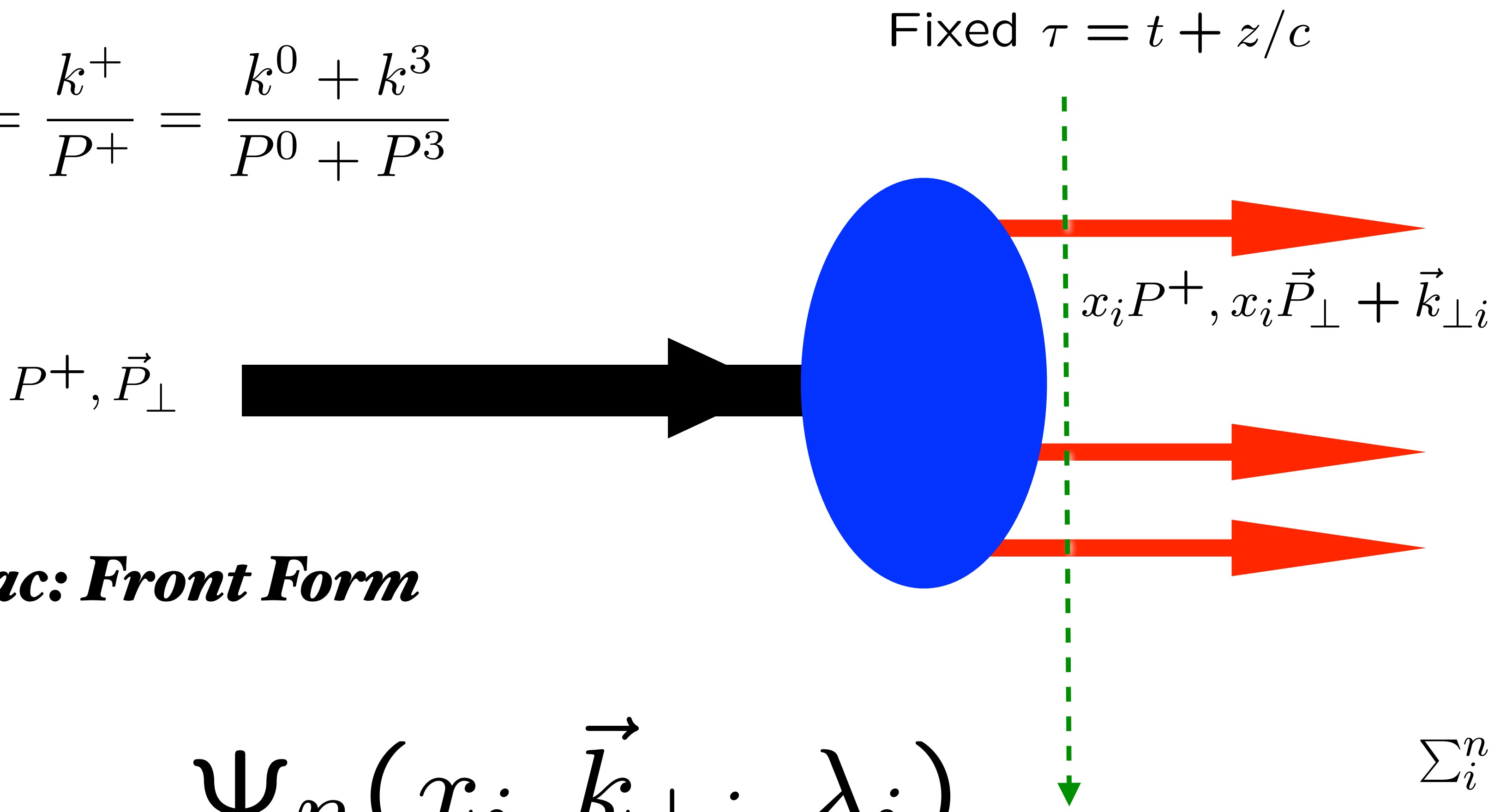
$\tau = t + z/c$

Instant Form

Front Form

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

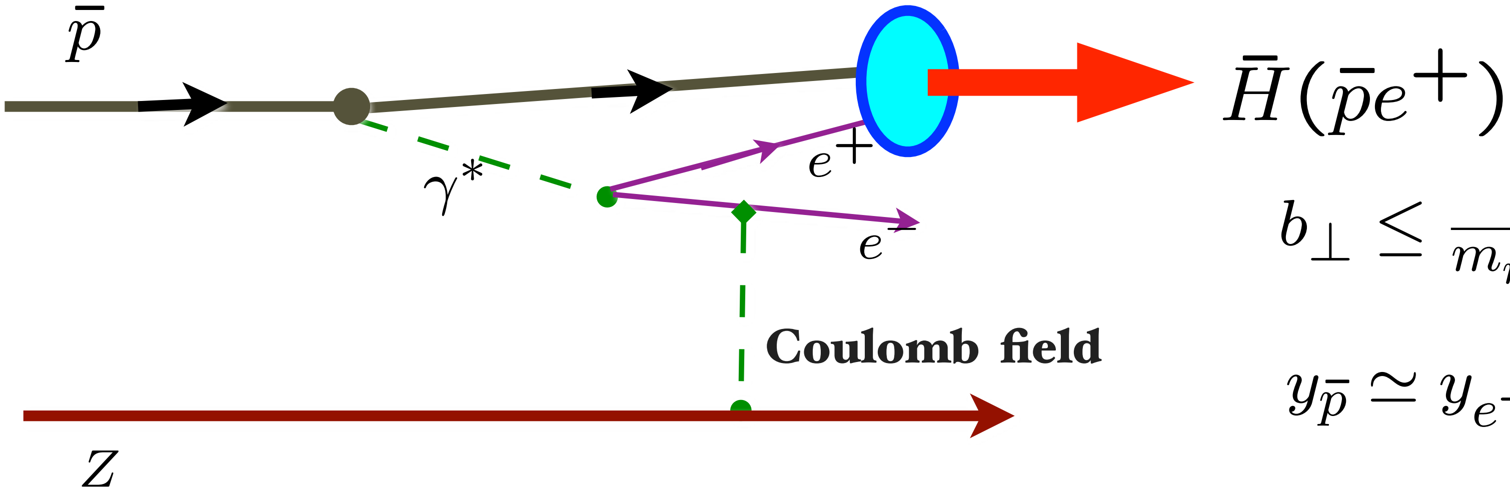
Invariant under boosts! Independent of P^μ

Causal, Frame-independent, Simple Vacuum, Current Matrix Elements are overlap of LFWFS

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

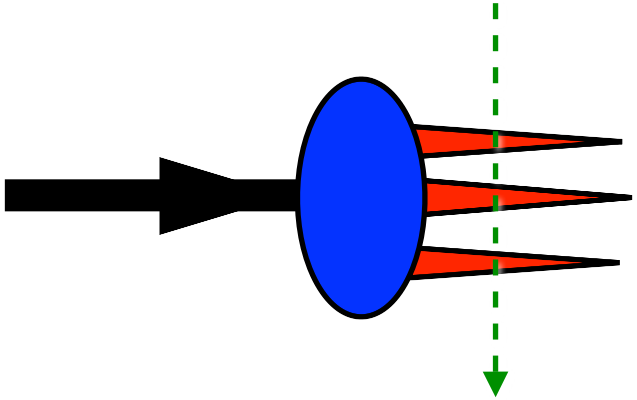
Coalescence of off-shell *co-moving positron and antiproton*

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

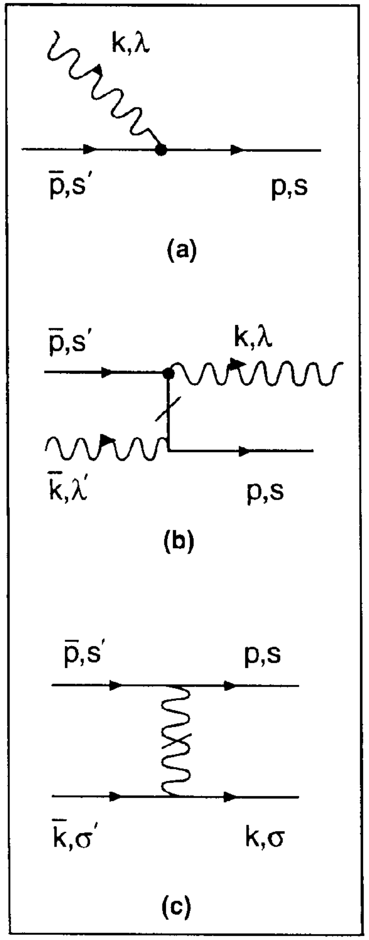
$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

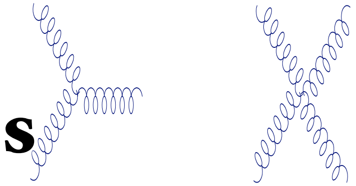
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



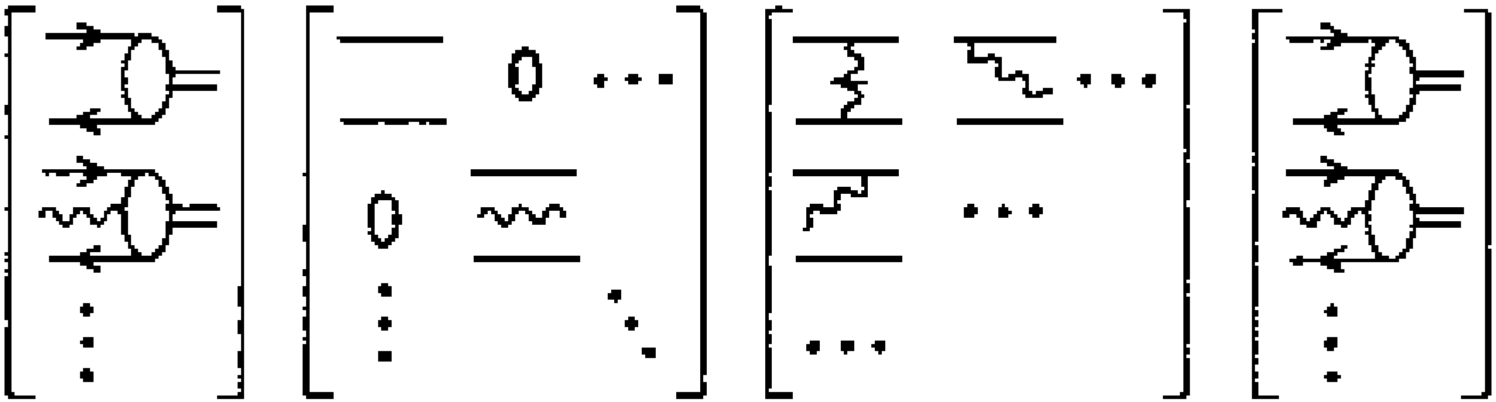
LFWFs: Off-shell in P- and invariant mass



$$H^{int}_{LF}$$

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

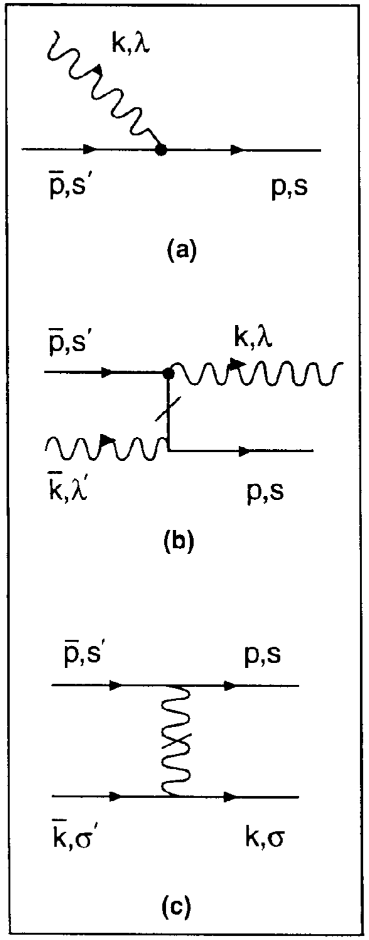
Light-Front QCD

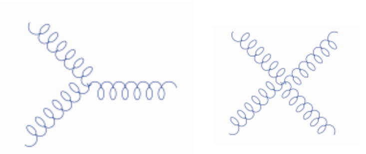
Heisenberg Equation

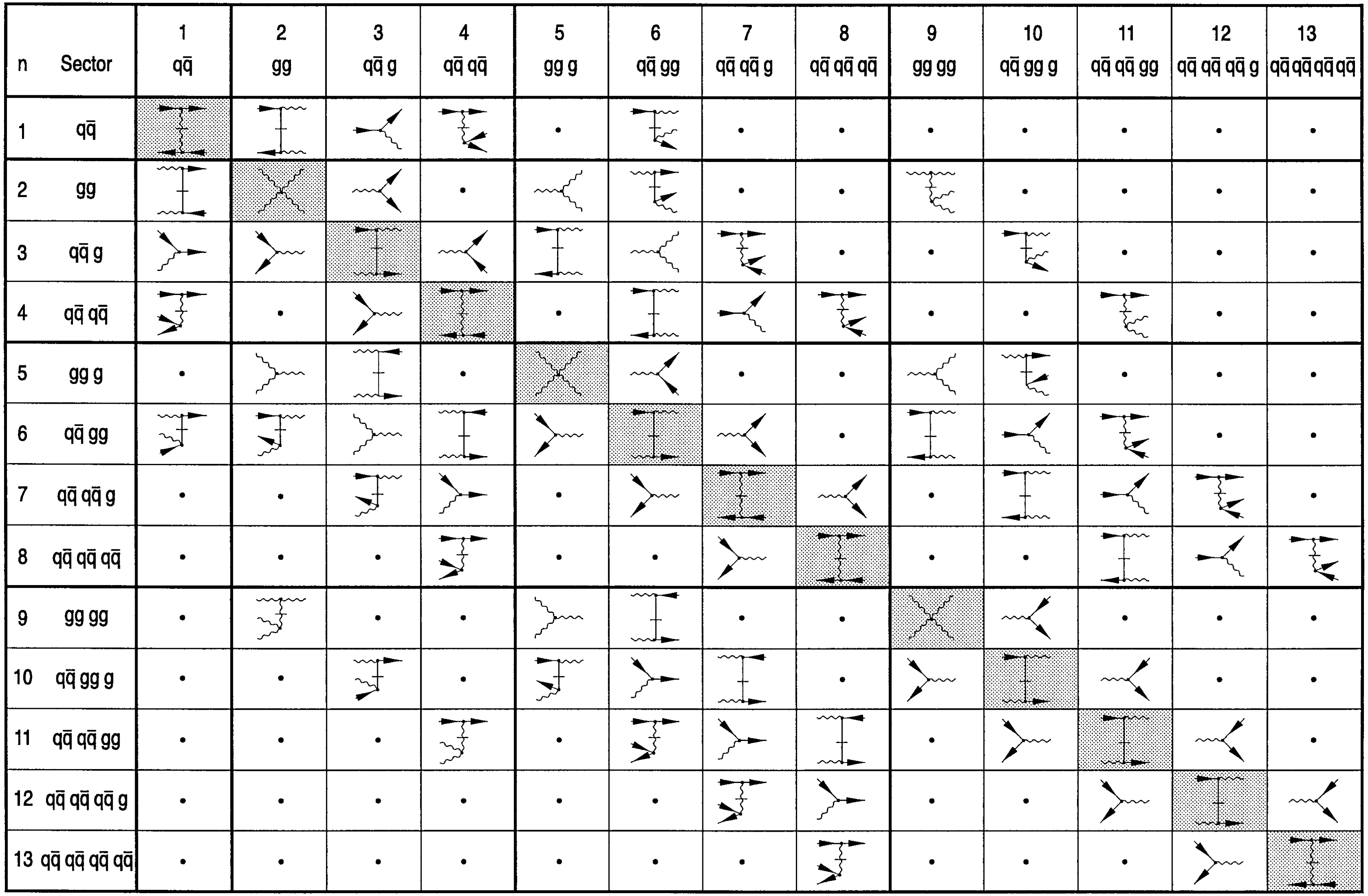
$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\,|\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







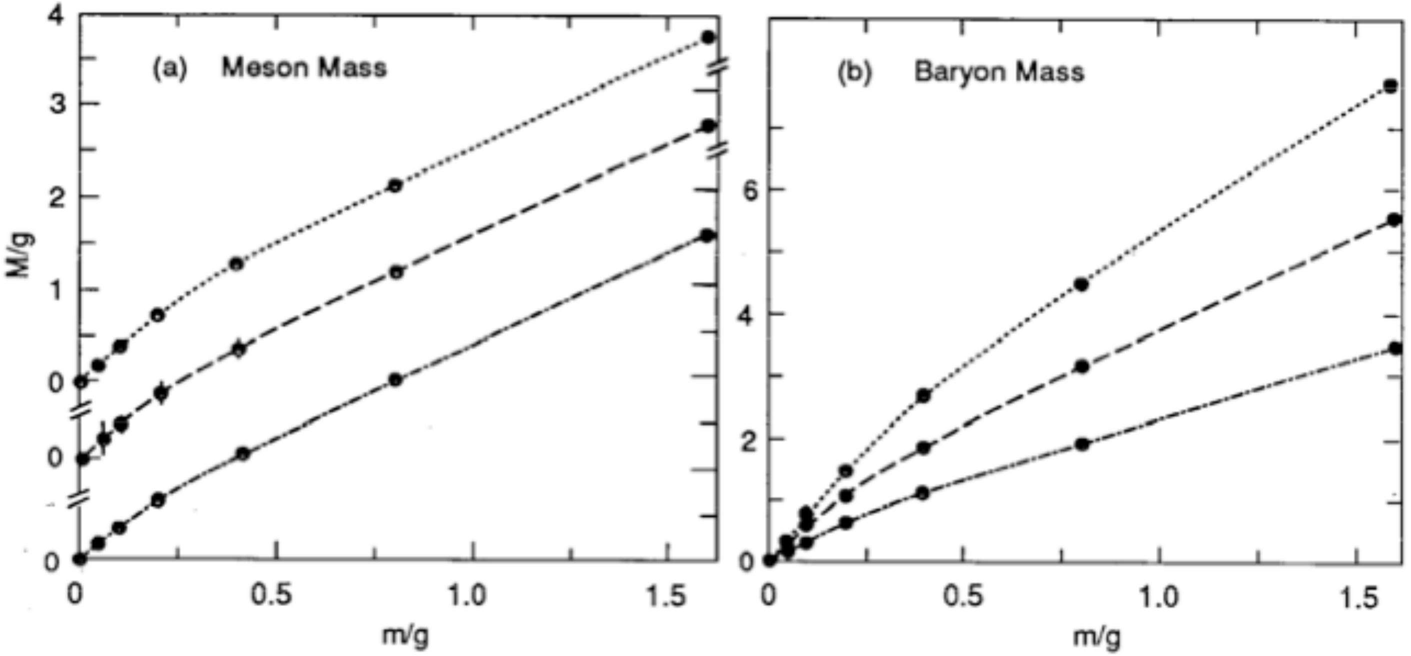
n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}\,g$	4 $q\bar{q}\,q\bar{q}$	5 $gg\,g$	6 $q\bar{q}\,gg$	7 $q\bar{q}\,q\bar{q}\,g$	8 $q\bar{q}\,q\bar{q}\,q\bar{q}$	9 $gg\,gg$	10 $q\bar{q}\,gg\,g$	11 $q\bar{q}\,q\bar{q}\,gg$	12 $q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	13 $q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$
1	$q\bar{q}$					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	$q\bar{q}\,g$								•	•		•	•	•
4	$q\bar{q}\,q\bar{q}$		•			•				•	•		•	•
5	$gg\,g$	•			•			•	•			•	•	•
6	$q\bar{q}\,gg$								•				•	•
7	$q\bar{q}\,q\bar{q}\,g$	•	•			•				•				•
8	$q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•		•	•			•	•			
9	$gg\,gg$	•		•	•			•	•			•	•	•
10	$q\bar{q}\,gg\,g$	•	•		•				•				•	•
11	$q\bar{q}\,q\bar{q}\,gg$	•	•	•		•				•				•
12	$q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	•	•	•	•	•	•			•	•			
13	$q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•	•	•	•	•		•	•	•		

Minkowski space; frame-independent; no fermion doubling; no ghosts

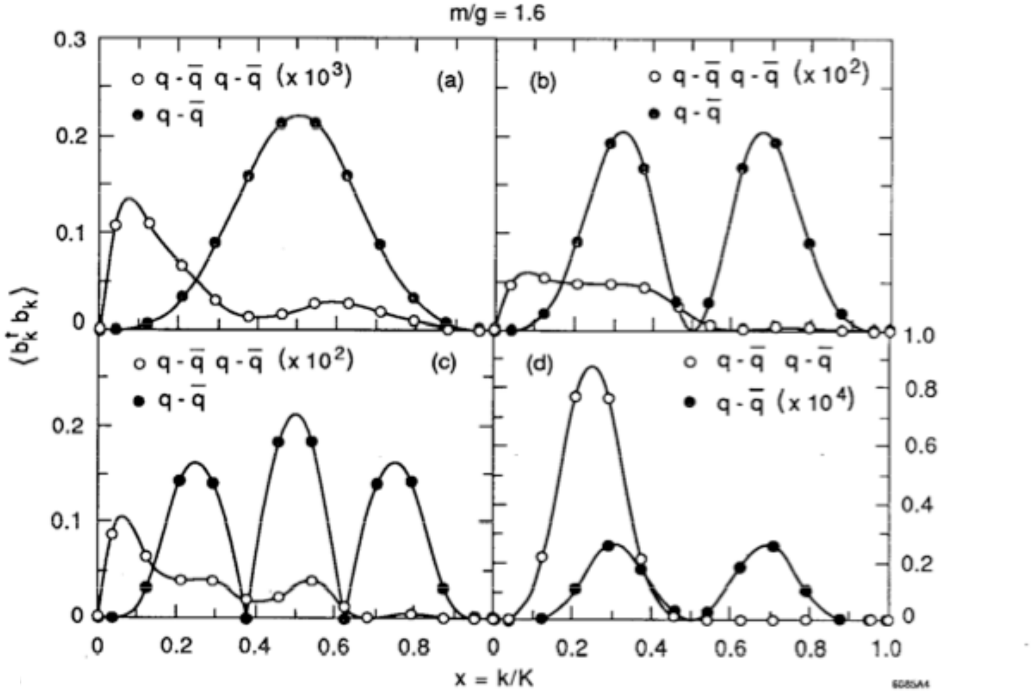
trivial vacuum

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

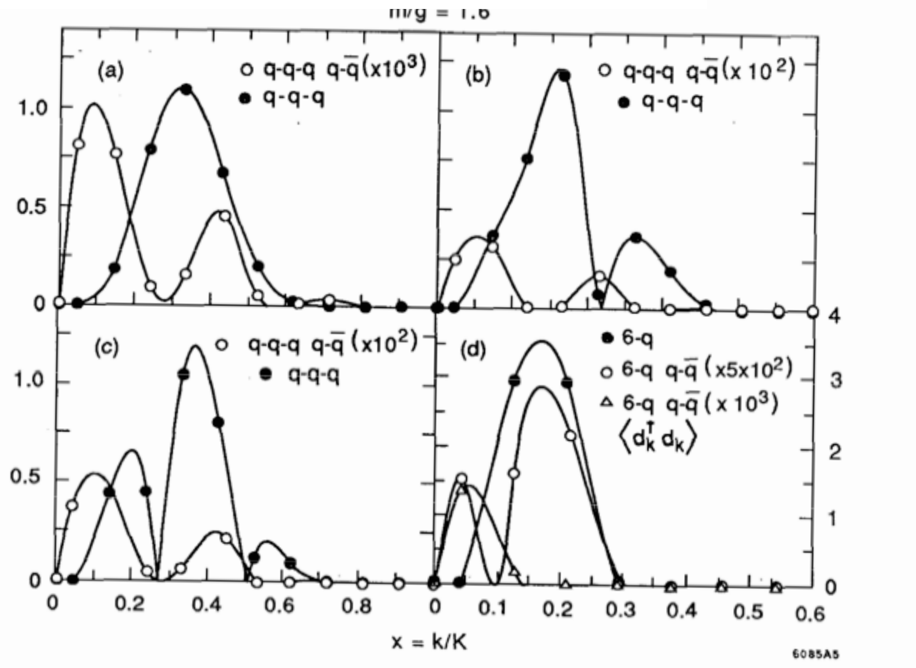
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.

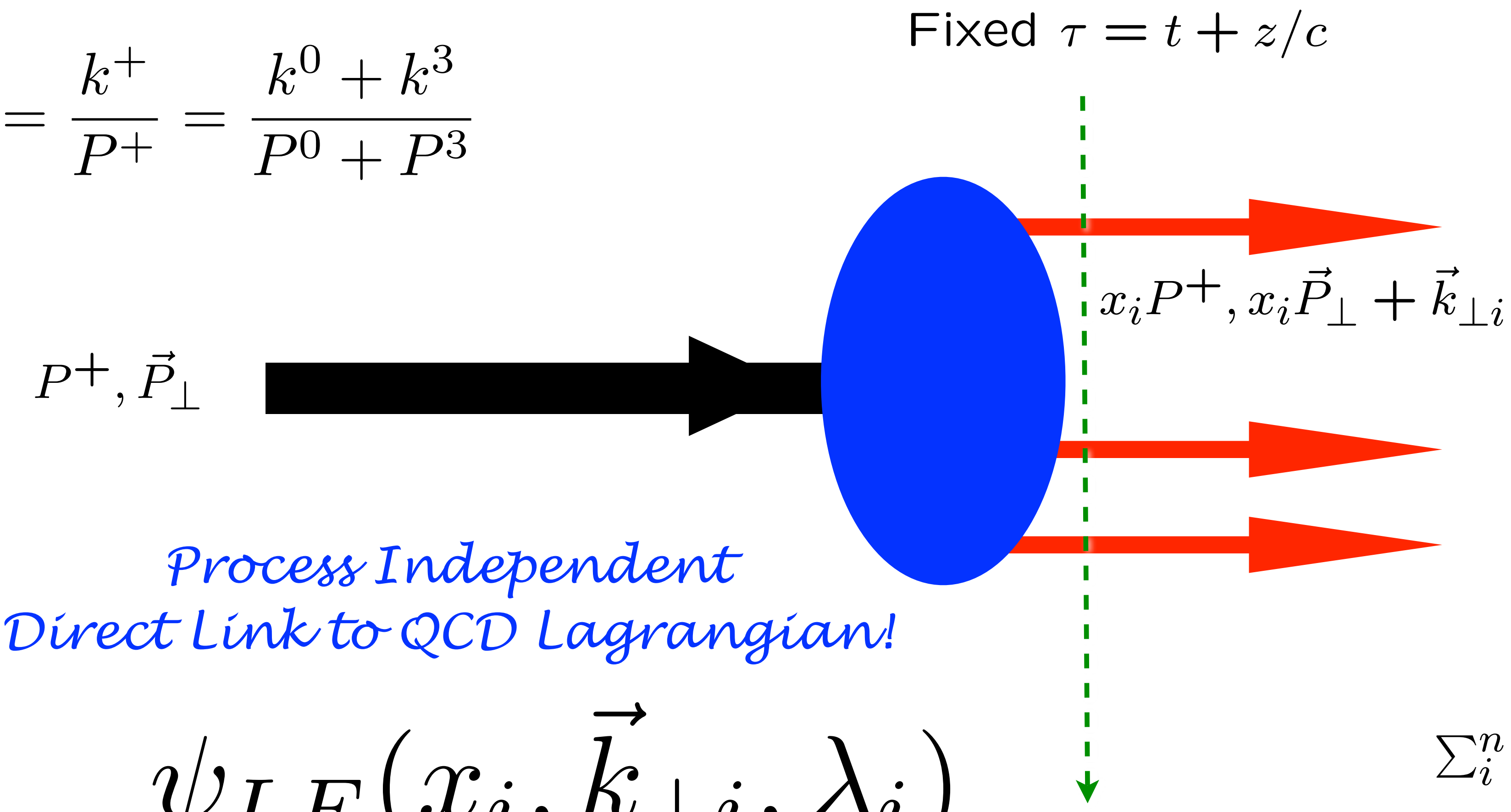


a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Process Independent
Direct Link to QCD Lagrangian!

$$\psi_{LF}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

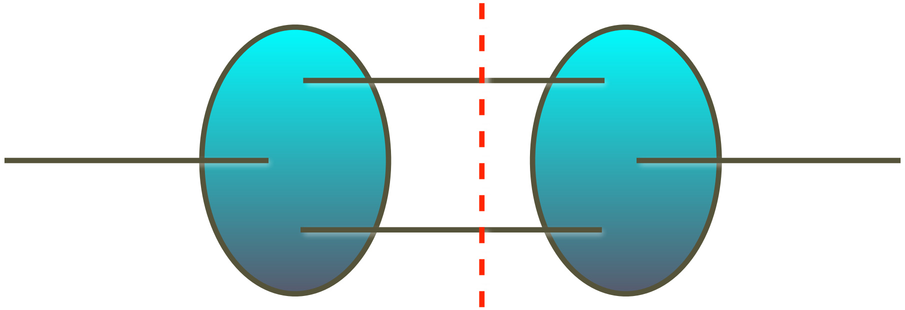
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

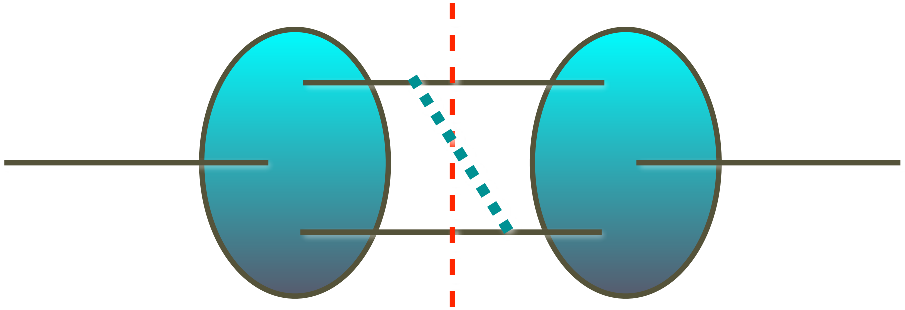
Quantum Mechanics: Uncertainty in p, x, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



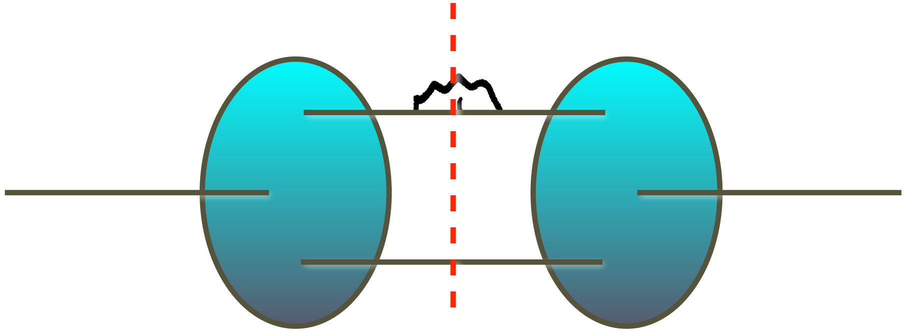
Positronium n=2

e^+e^-



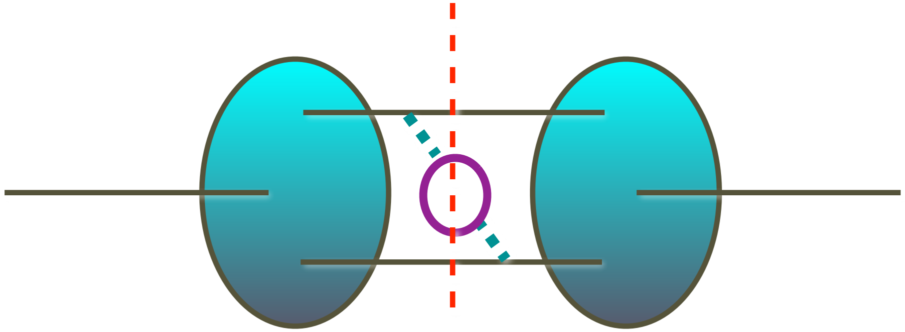
Hyperfine splitting n=3

$e^+e^-\gamma$



Lamb Shift n=3

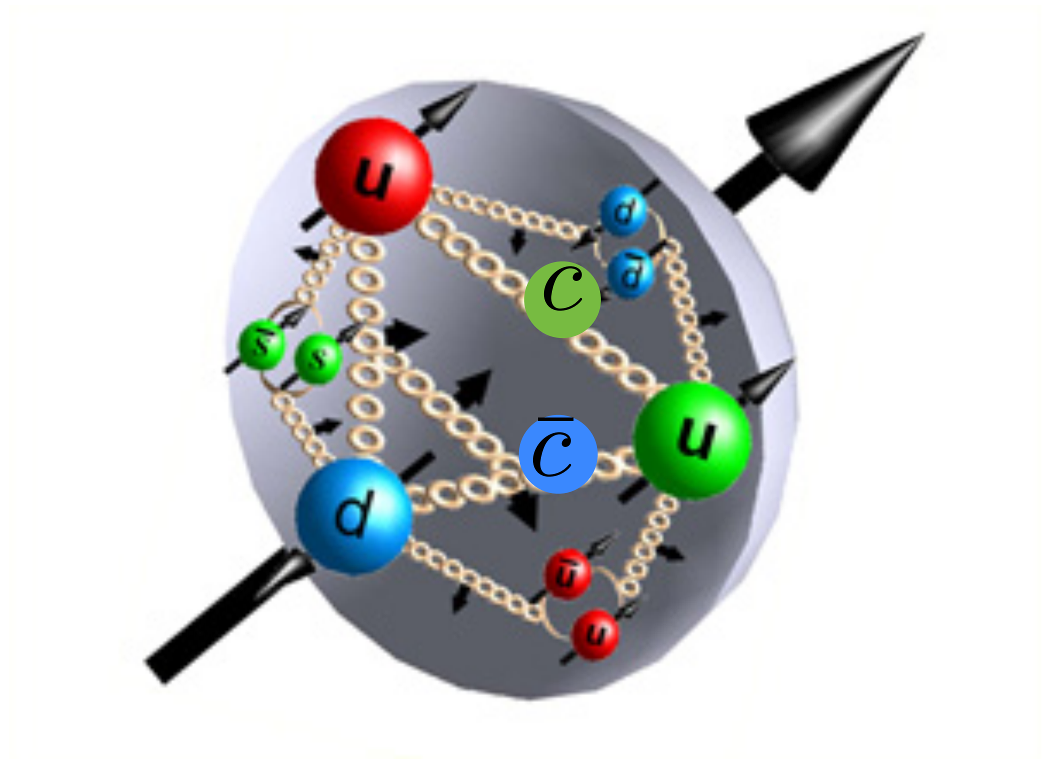
$e^+e^-\gamma$

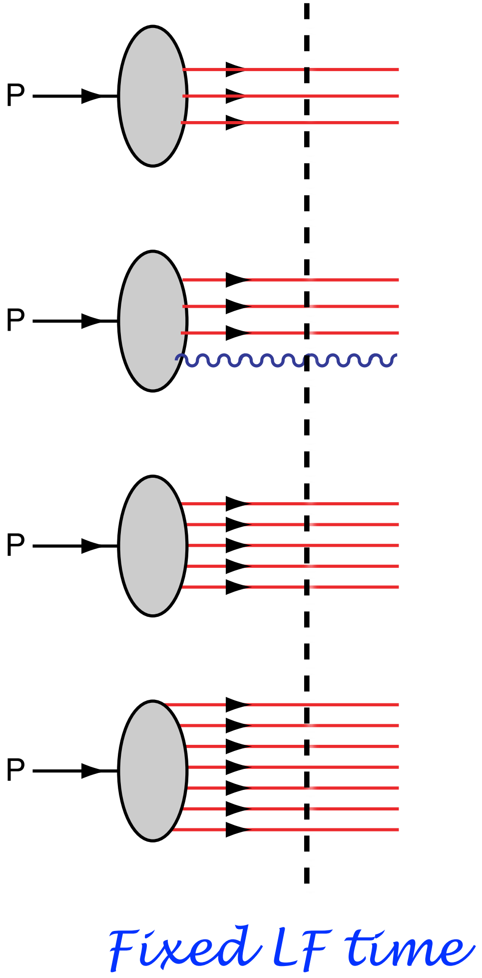


Vacuum Polarization n=4

$e^+e^-e^+e^-$

Higher Fock States of the Proton

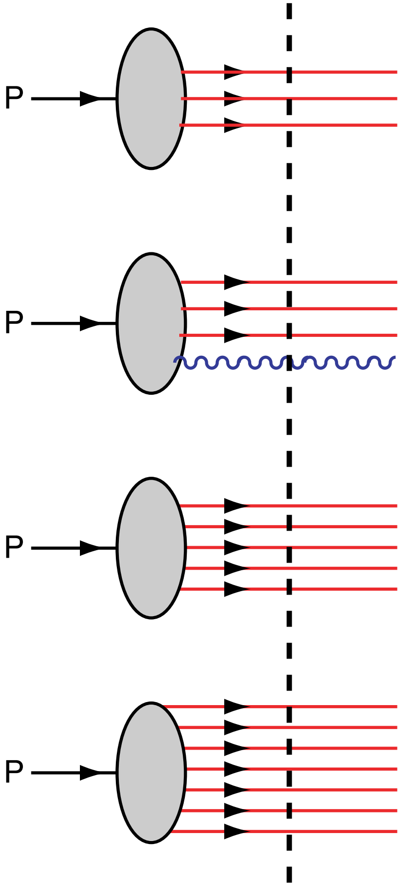




Fixed LF time: Off-Shell in invariant mass
Quantum Field Theory: Higher Fock States

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents



The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks

c(x), b(x) at high x !

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Fixed LF time

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high Q^2

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock
State!

LF Spin Sum Rule

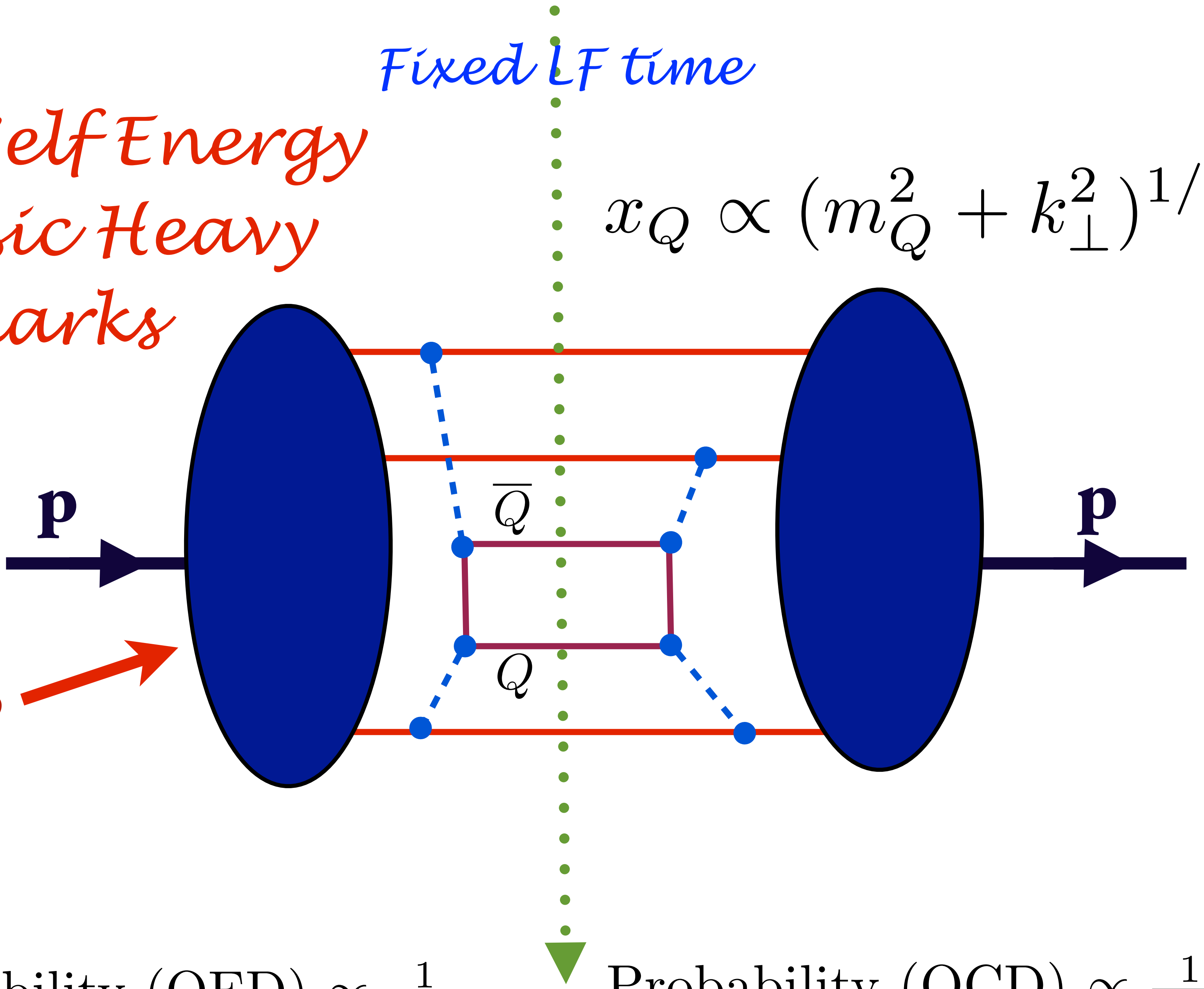
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Orbital angular momentum is a property of Light-Front Wavefunctions

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Proton Self Energy Intrinsic Heavy Quarks



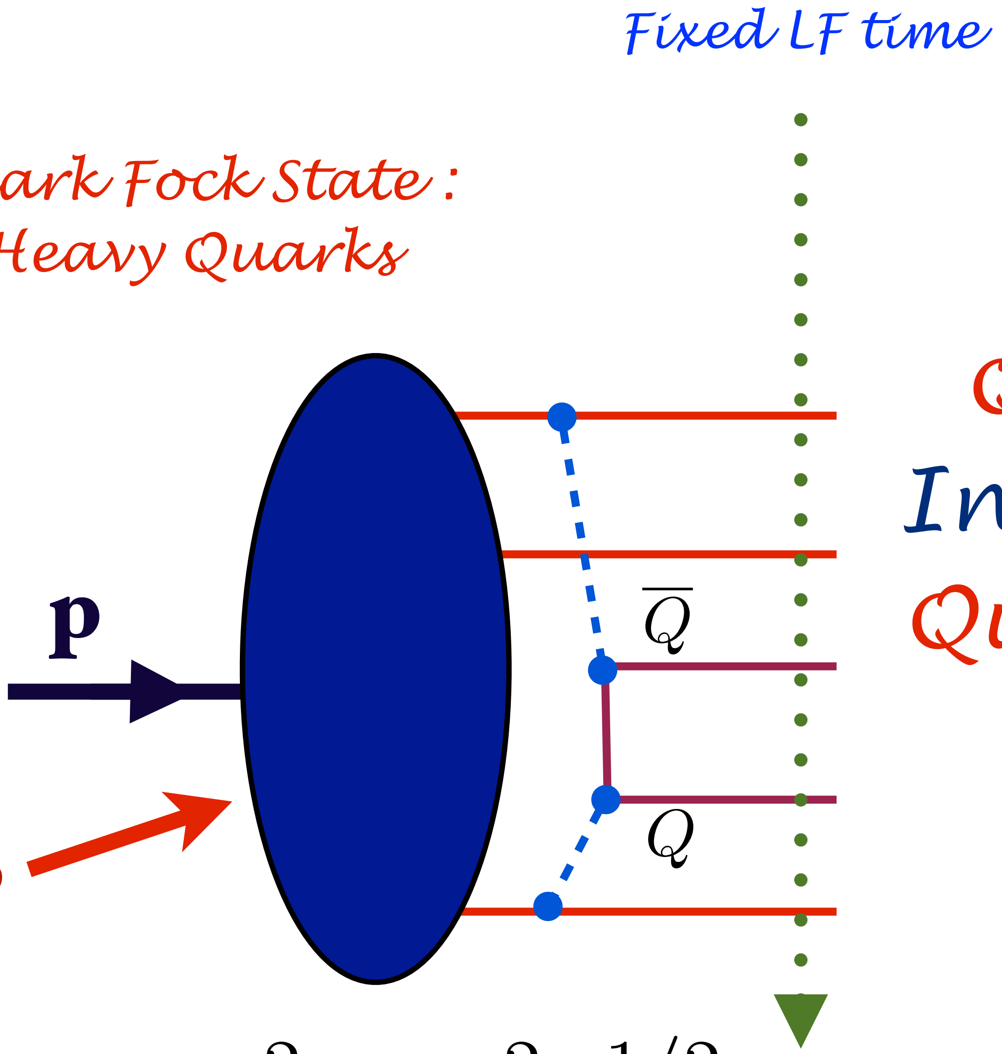
$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

Use AdS/QCD LFWF

$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$ $\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

Proton 5-quark Fock State : Intrinsic Heavy Quarks



Use AdS/QCD LFWF

QCD predicts Intrinsic Heavy Quarks at high x!

Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

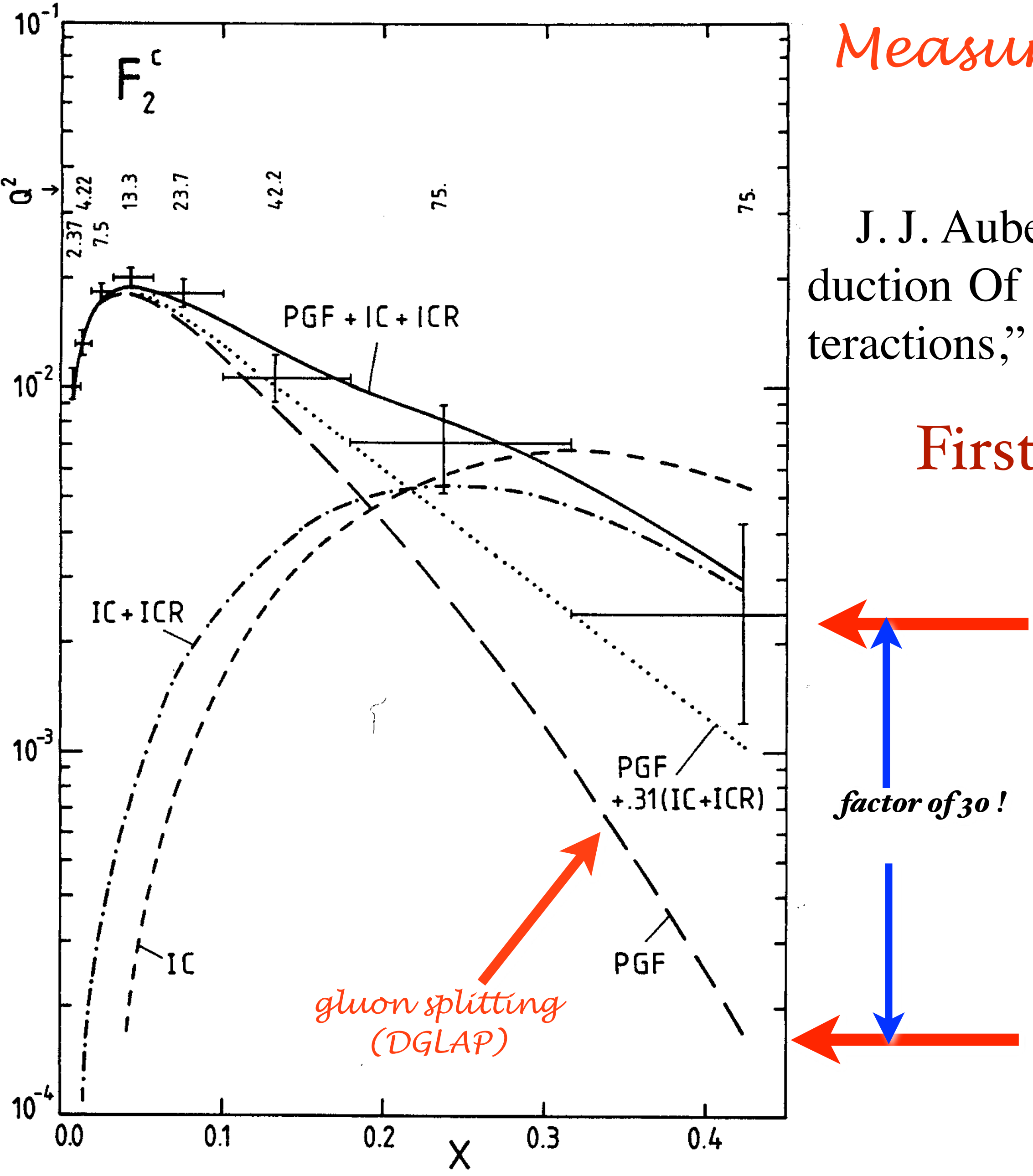
$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

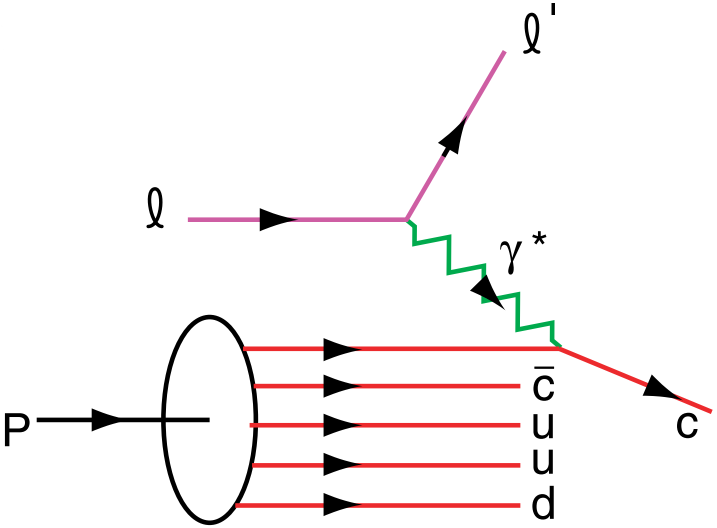
Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], “Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions,” Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm





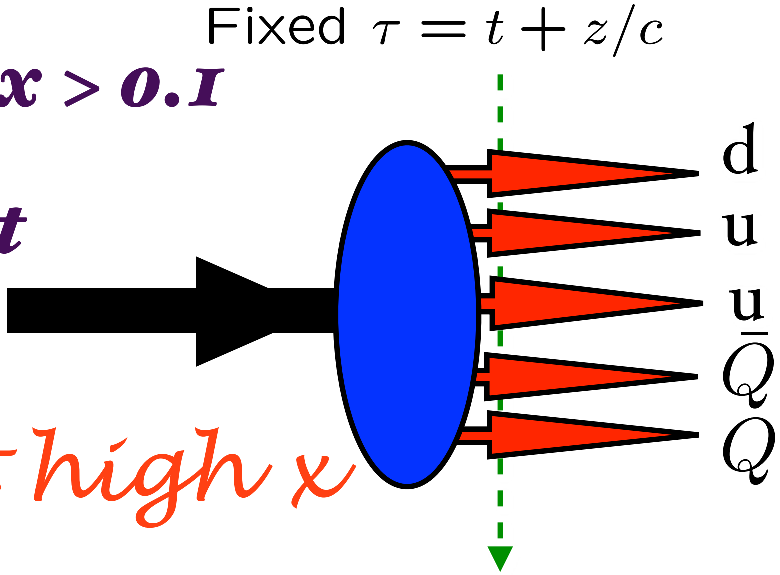
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x,Q^2) = c(x,Q^2)_{\text{extrinsic}} + c(x,Q^2)_{\text{intrinsic}}$$

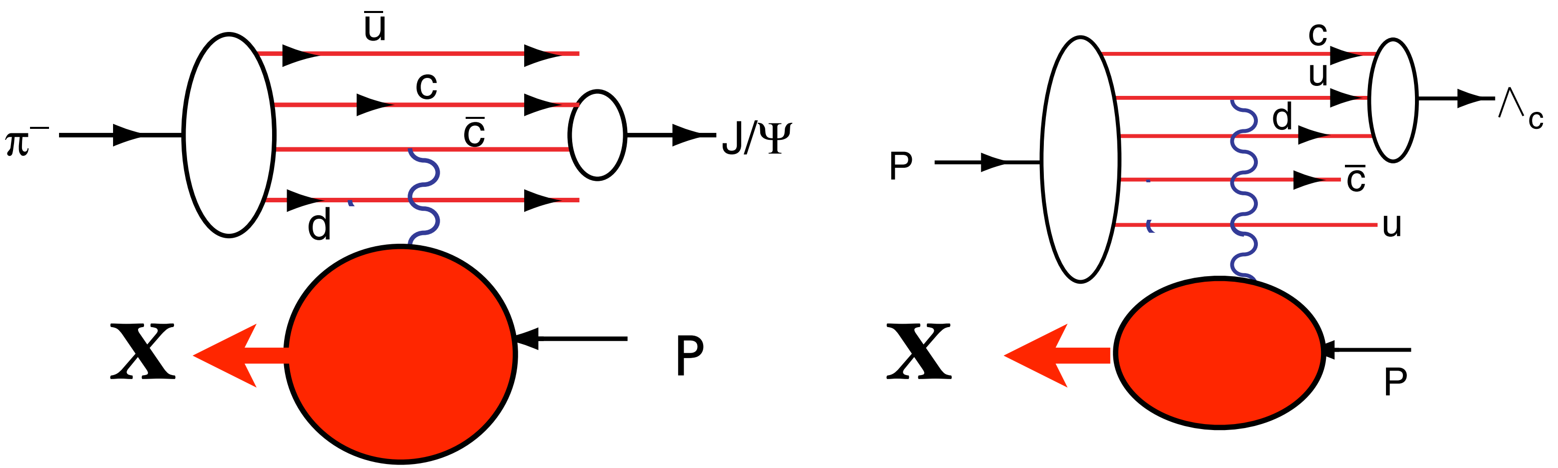
Properties of Non-Perturbative 5 and 7 -Quark Fock-State

- ***Dominant configuration: same rapidity***
- ***Heavy quarks have most momentum***
- ***Correlated with proton quantum numbers***
- ***Duality with meson-baryon channels***
- ***strangeness asymmetry at x > 0.1***
- ***Maximally energy efficient***



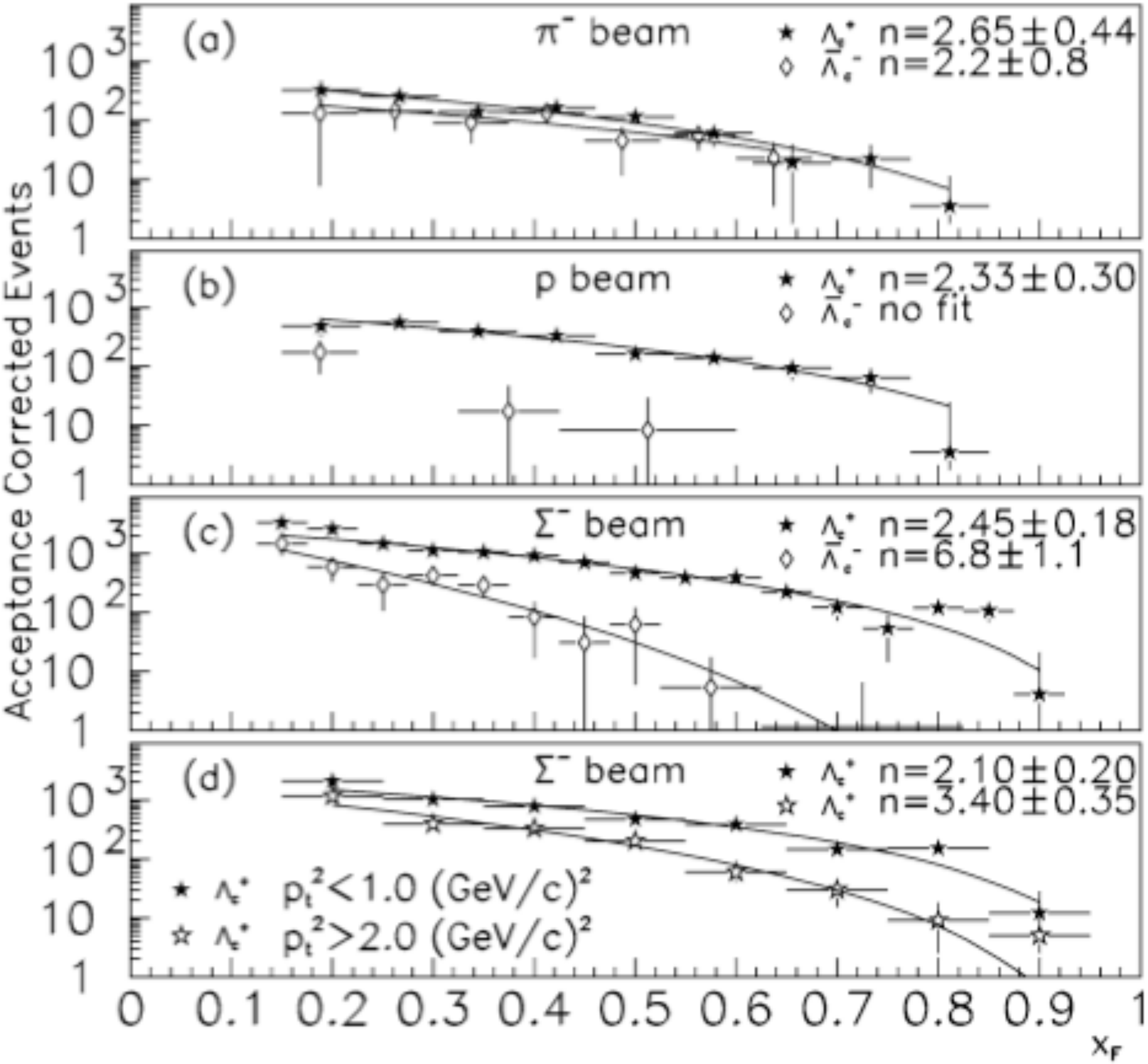
Intrinsic Heavy Quarks at high x

Leading Hadron Production from "Intrinsic Charm"

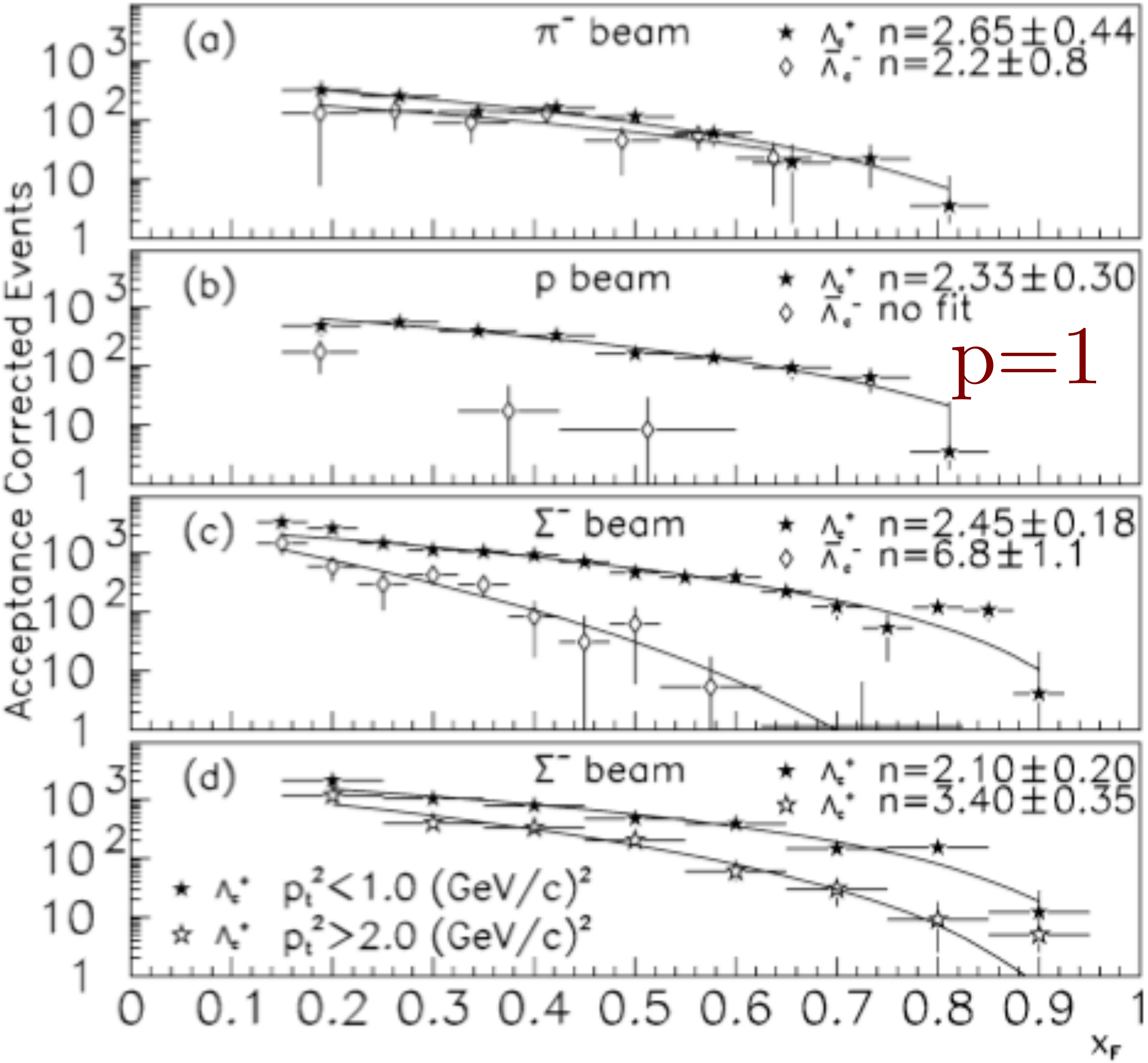


Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F

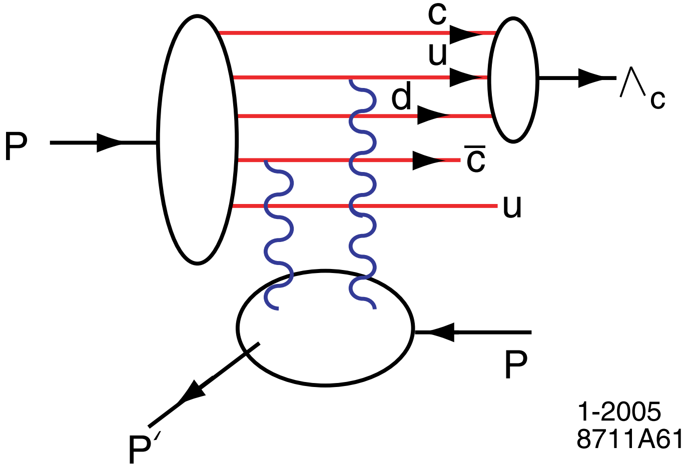
SELEX Collaboration / Physics Letters B 528 (2002) 49–57



Large xF production close to the maximum allowed by phase space!



p=1



$$p(uudc\bar{c})$$

$$\rightarrow \Lambda_c(cud)$$

$$n_s = 2$$

Phase space gives minimum power p

$$(1 - x_F)^p, p = n_s - 1$$

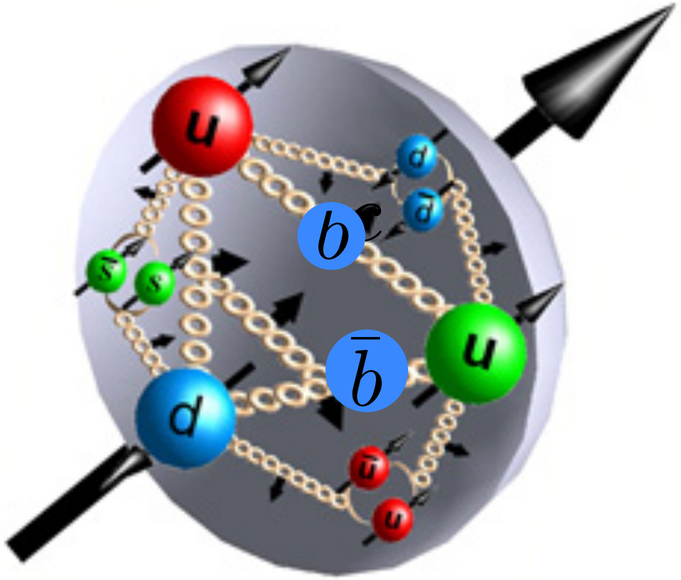
27 May 1991

CM-P00063074

THE $\Lambda_b{}^o$ BEAUTY BARYON PRODUCTION IN PROTON-PROTON INTERACTIONS AT $\sqrt{s}$=62 GeV: A SECOND OBSERVATION

G. Bari, M. Basile, G. Bruni, G. Cara Romeo, R. Casaccia, L. Cifarelli,
F. Cindolo, A. Contin, G. D'Alì, C. Del Papa, S. De Pasquale, P. Giusti,
G. Iacobucci, G. Maccarrone, T. Massam, R. Nania, F. Palmonari,
G. Sartorelli, G. Susinno, L. Votano and A. Zichichi

CERN, Geneva, Switzerland
Dipartimento di Fisica dell'Università, Bologna, Italy
Dipartimento di Fisica dell'Università, Cosenza, Italy
Istituto di Fisica dell'Università, Palermo, Italy
Istituto Nazionale di Fisica Nucleare, Bologna, Italy
Istituto Nazionale di Fisica Nucleare, LNF, Frascati, Italy



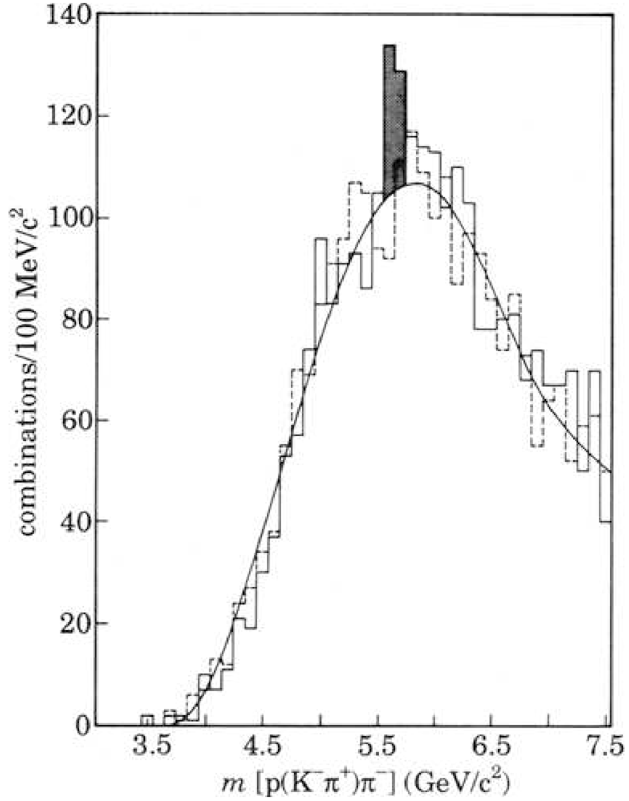
Abstract

Another decay mode of the $\Lambda_b{}^o$ (open-beauty baryon) state has been observed: $\Lambda_b{}^o \rightarrow \Lambda_c{}^+\pi^+\pi^-\pi^-$. In addition, new results on the previously observed decay channel, $\Lambda_b{}^o \rightarrow pD^o\pi^-$, are reported. These results confirm our previous findings on $\Lambda_b{}^o$ production at the ISR. The mass value (5.6 GeV/c^2) is found to be in good agreement with theoretical predictions. The production mechanism is found to be "leading".

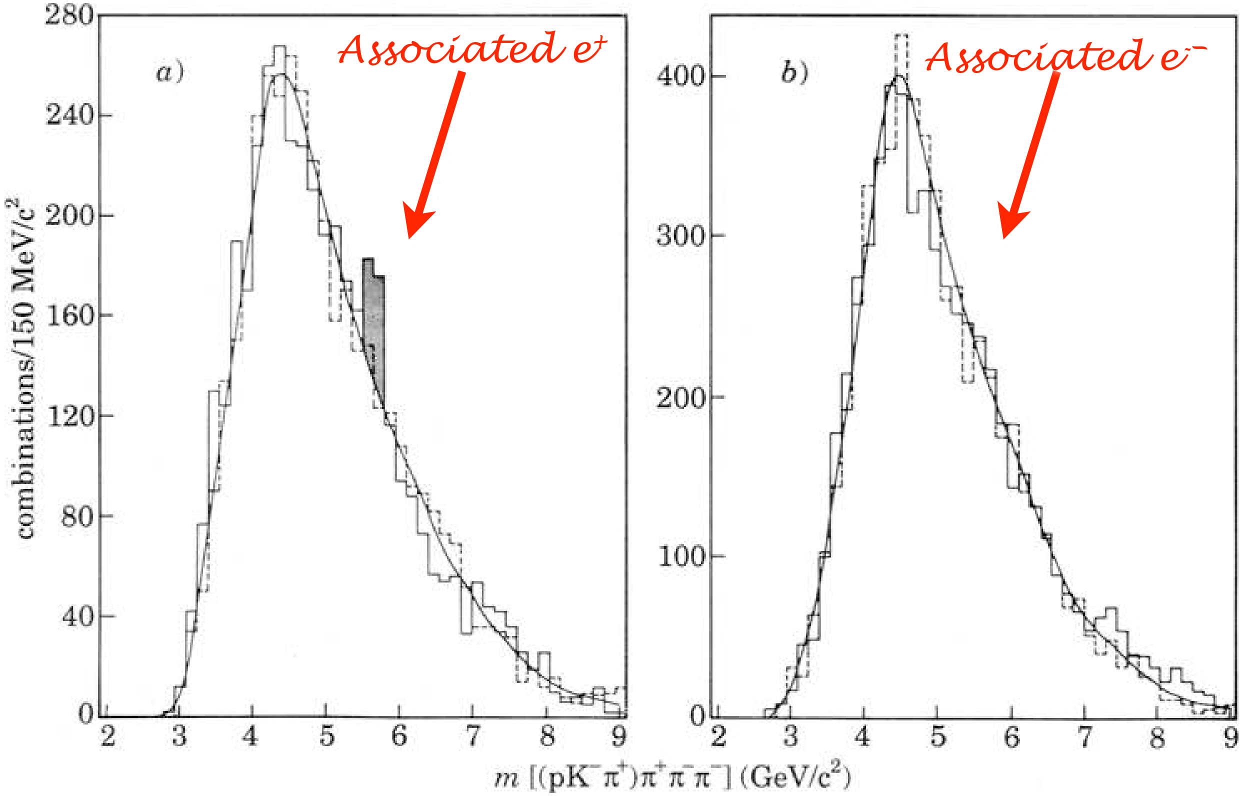
Evidence for Intrinsic Bottom!

$pp \rightarrow \Lambda_b(bud)B(\bar{b}q)X$ at large x_F

CERN-ISR R422 (Split Field Magnet), 1988/1991



$\Lambda_b^0 \rightarrow pD^0\pi^-$

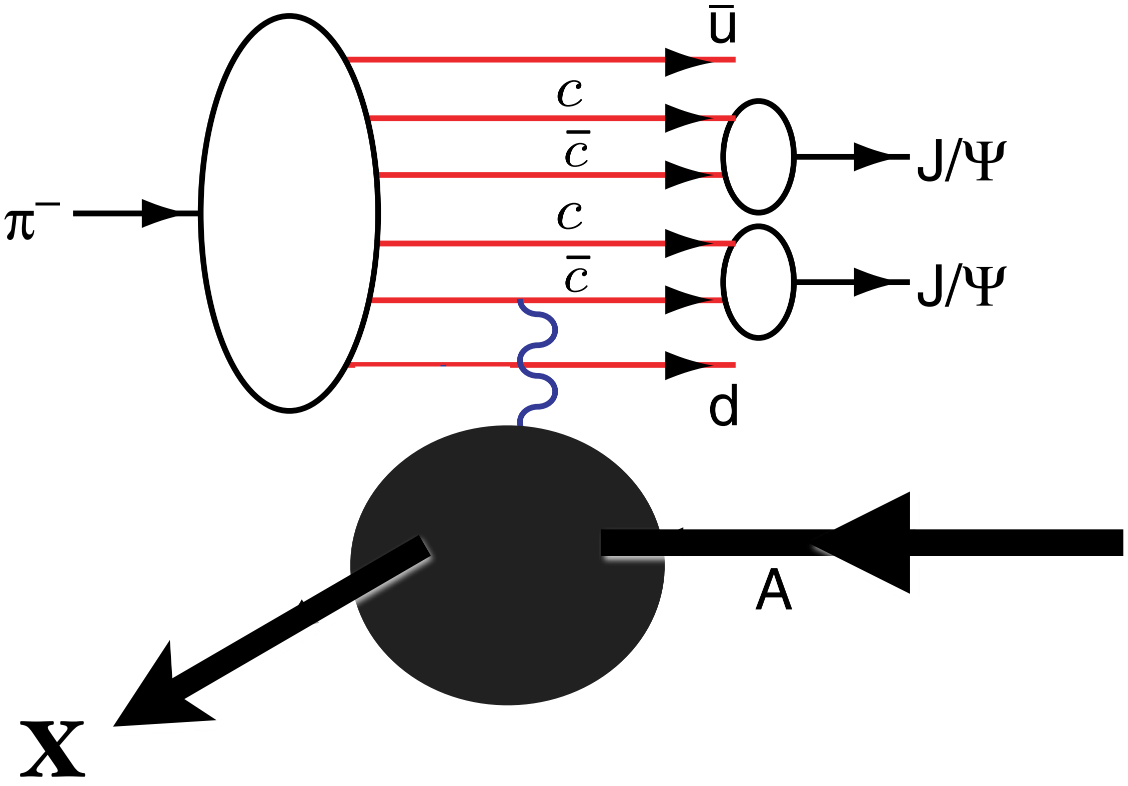


$\Lambda_b^0 \rightarrow \Lambda_c^+\pi^+\pi^-\pi^-$

Il Nuovo Cimento 104, 1787

Evidence for Intrinsic Bottom!

Production of Two Charmonia at High x_F



Excludes PYTHIA ‘color drag’ model

All events have $x^F_{\psi\psi} > 0.4$!

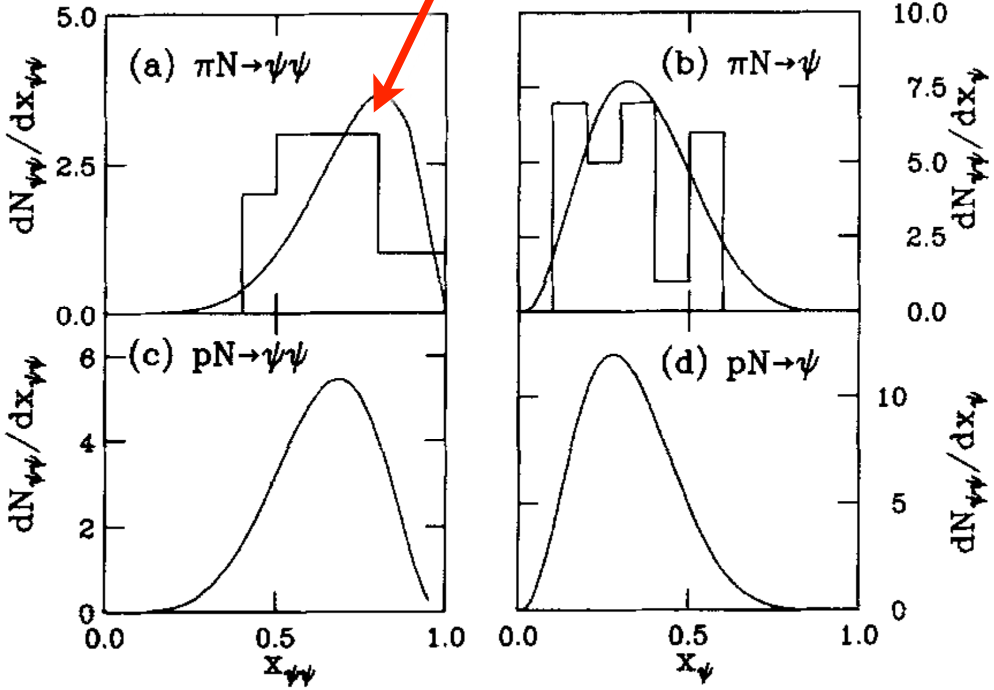


Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of J/ψ’s from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/c [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single J/ψ’s is twice the number of pairs.

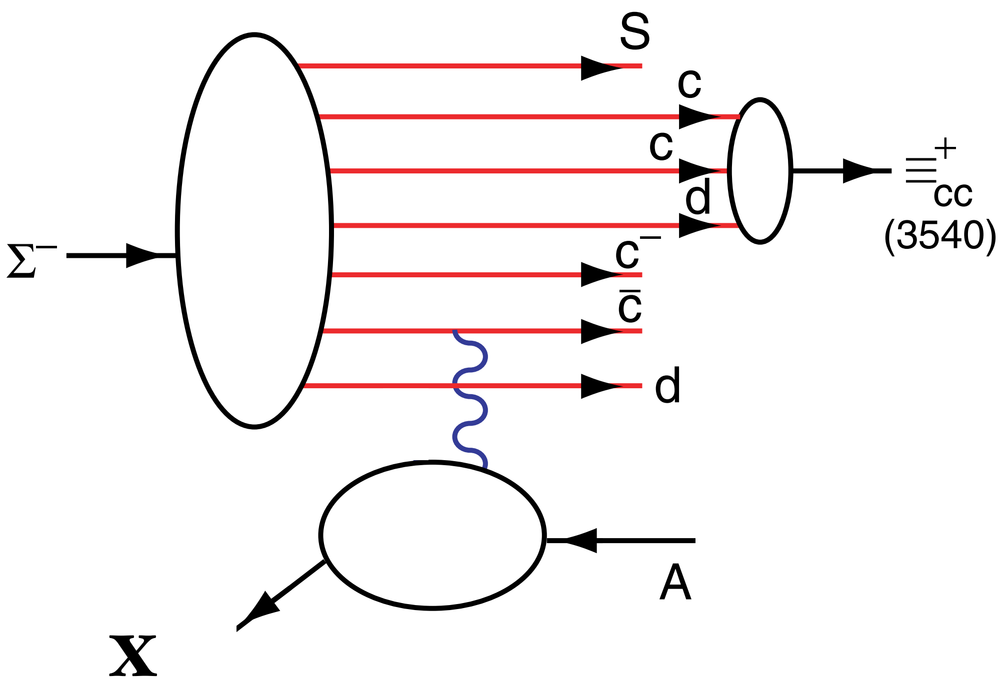
NA3 Data

$$\pi A \rightarrow J/\psi J/\psi X$$

R. Vogt, sjb

The probability distribution for a general n-particle intrinsic $c\bar{c}$ Fock state as a function of x and $\boldsymbol{k}_T$ is written as

$$\frac{dP_{\rm ic}}{\prod_{i=1}^{n} dx_i d^2 k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^{n} \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^{n} x_i)}{(m_h^2 - \sum_{i=1}^{n}(m_{T,i}^2/x_i))^2} \, ,$$

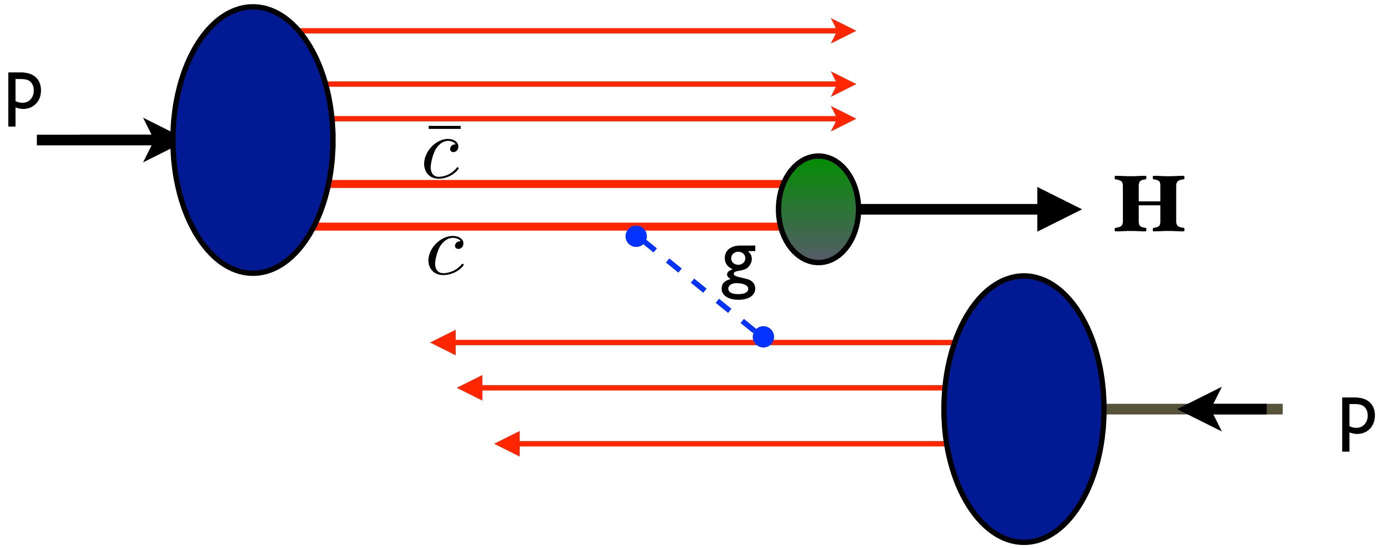


Production of a Double-Charm Baryon

SELEX high x_F $< x_F >= 0.33$

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \rightarrow H X$$

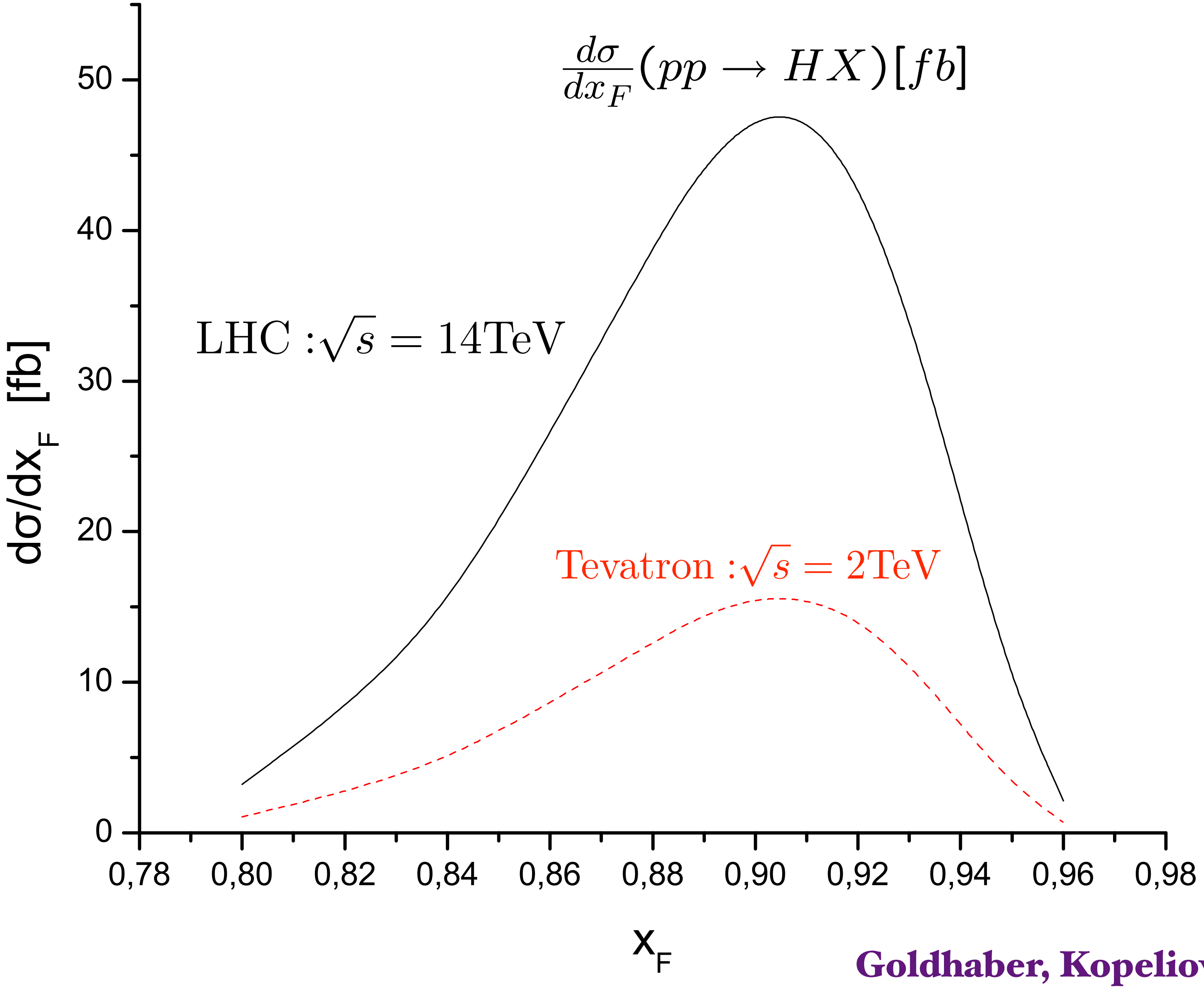
Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs!

AFTER: Higgs production at threshold!

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production

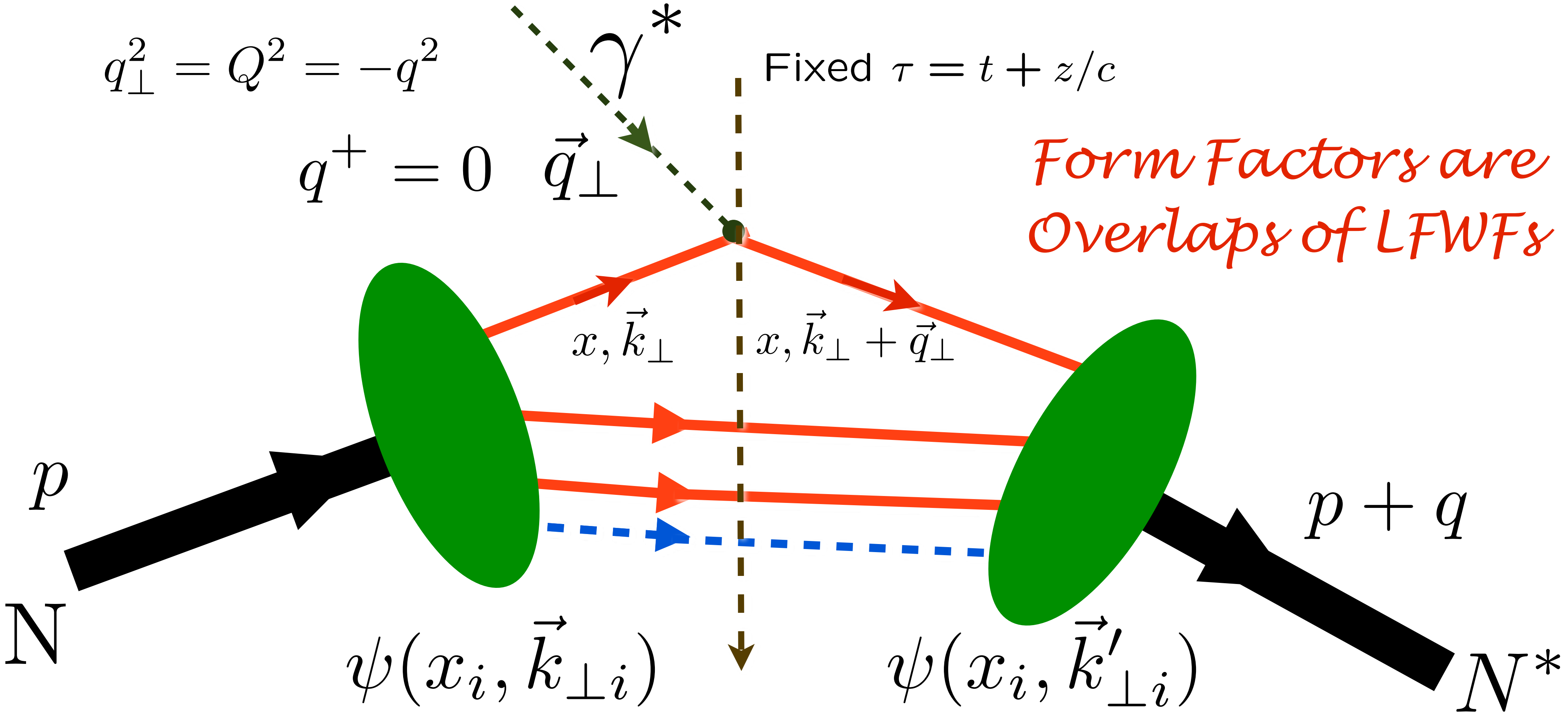


Goldhaber, Kopeliovich, Schmidt, sjb

Engelfried & SJB:
Detect 4 muon and 2 muon final states at LHC downstream

$$< p + q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Drell, sjb
Exact LF formula
Sum over Fock states

Exact LF Formula for Pauli Form Factor

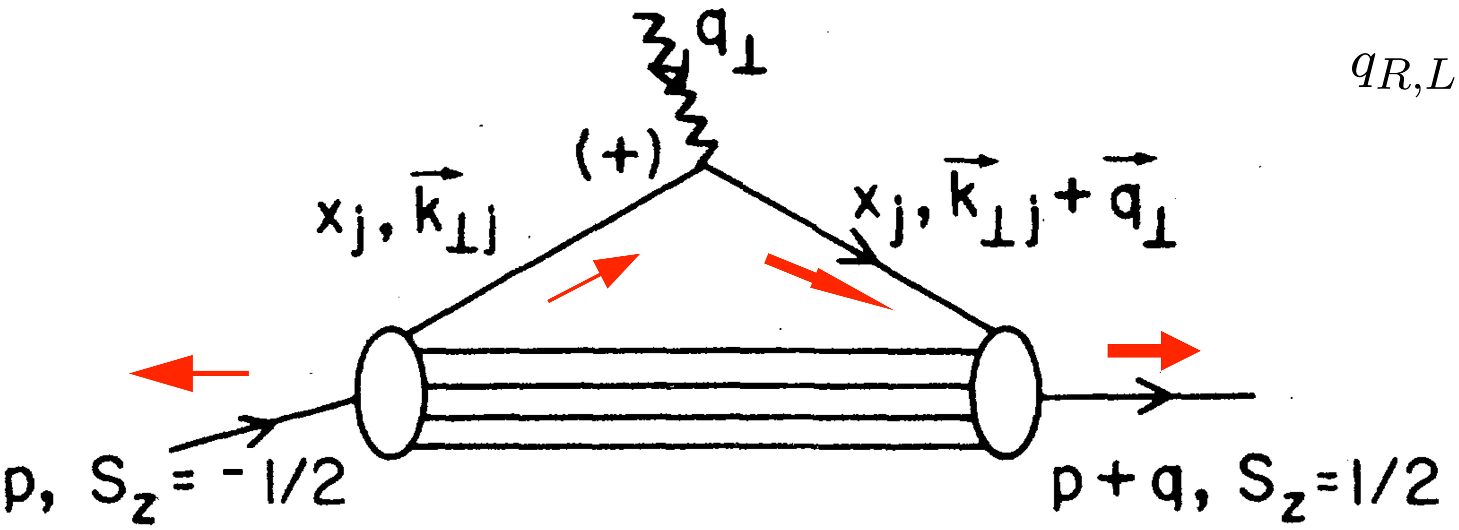
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Novel World of Hadron Physics

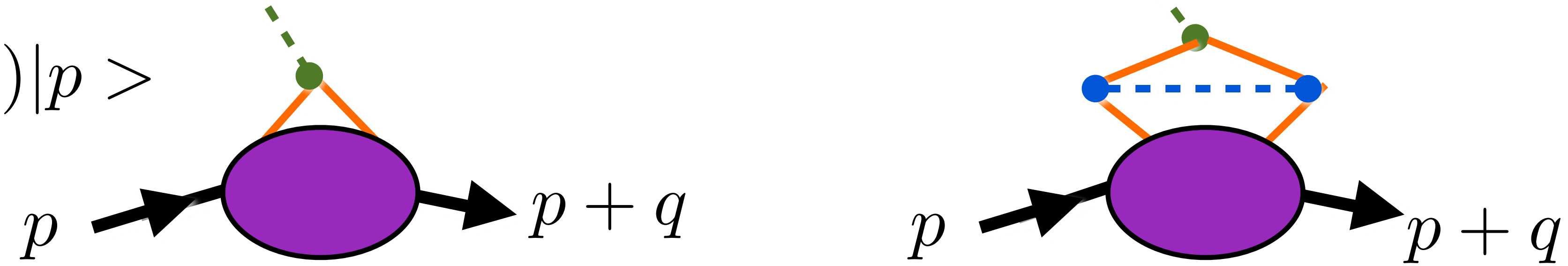
UNIVERSITY of VIRGINIA

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Calculation of proton form factor in Instant Form

$< p+q|J^{\mu}(0)|p >$



- **Need to boost proton wavefunction: p to p+q. Extremely complicated dynamical problem; particle number changes**

- **Need to couple to all currents arising from vacuum!! Remain even after normal-ordering**

- **Instant-form WFs insufficient to calculate form factors**

- **Each time-ordered contribution is frame-dependent**

- **Divide by disconnected vacuum diagrams**

Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**
- **Frame-Independent, Causal**
- **Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive**
- **J^z conserved at each vertex**
- **Cluster Decomposition -- only proof for relativistic theory**
- **Automatically normal-ordered; LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto-Cruz)**
- **Hadronization at the Amplitude Level with Confinement**

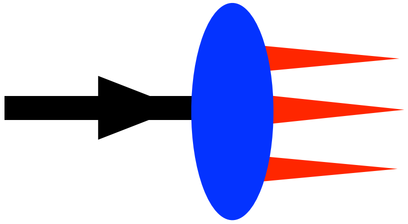
- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**

- **Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!**



$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- *Hadron Physics without LFWFs is like Biology without DNA!*

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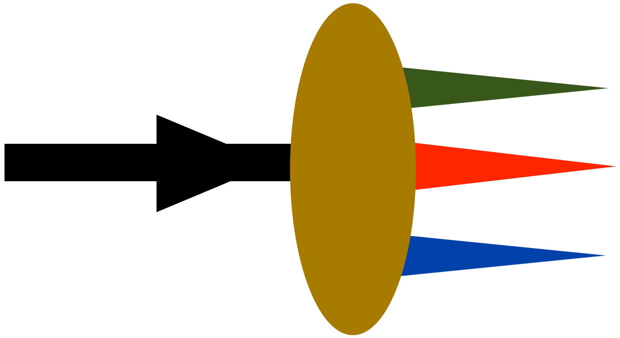
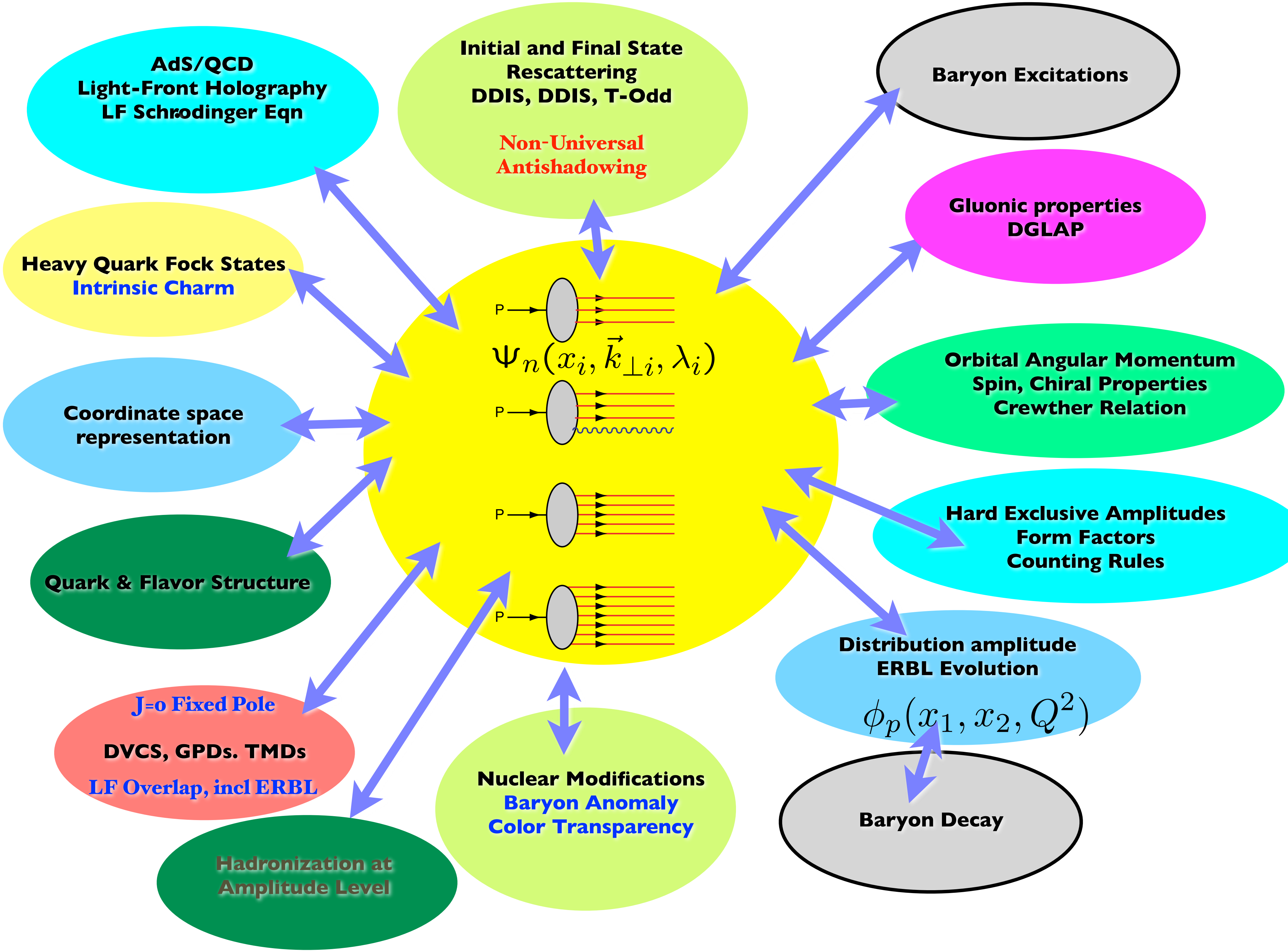


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

QCD and the LF Hadron Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

AdS/QCD
Light-Front Holography
LF Schrodinger Eqn

Initial and Final State
Rescattering
DDIS, DDIS, T-Odd

Non-Universal
Antishadowing

Baryon Excitations

Gluonic properties
DGLAP

Heavy Quark Fock States
Intrinsic Charm

Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation

Coordinate space
representation

Hard Exclusive Amplitudes
Form Factors
Counting Rules

Quark & Flavor Structure

Distribution amplitude
ERBL Evolution

$$\phi_p(x_1, x_2, Q^2)$$

J=0 Fixed Pole

DVCS, GPDs. TMDs

LF Overlap, incl ERBL

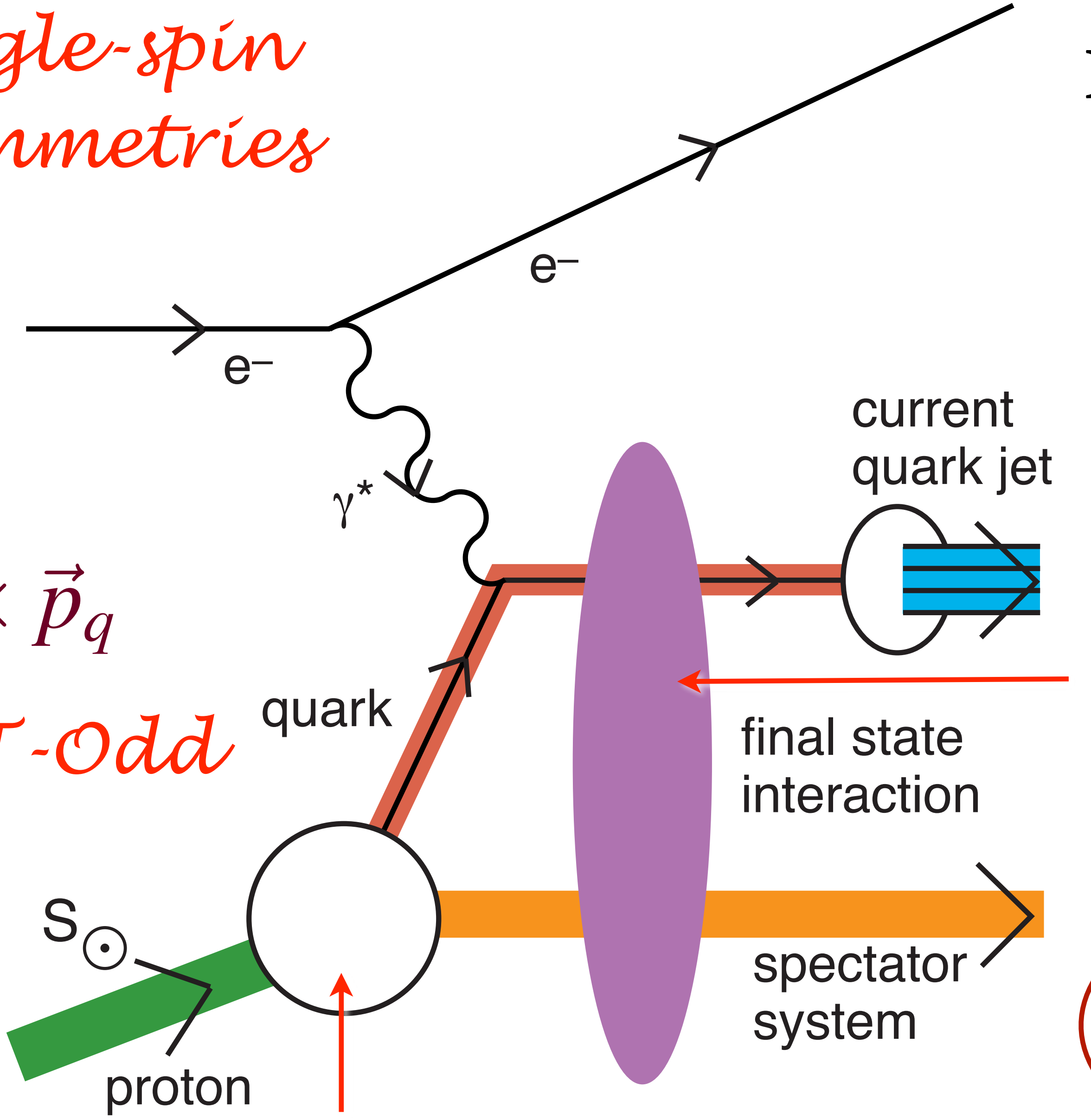
Nuclear Modifications
Baryon Anomaly
Color Transparency

Baryon Decay

Hadronization at
Amplitude Level

Leading-Twist Sivers Effect

Single-spin asymmetries



Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases --Wilson Line

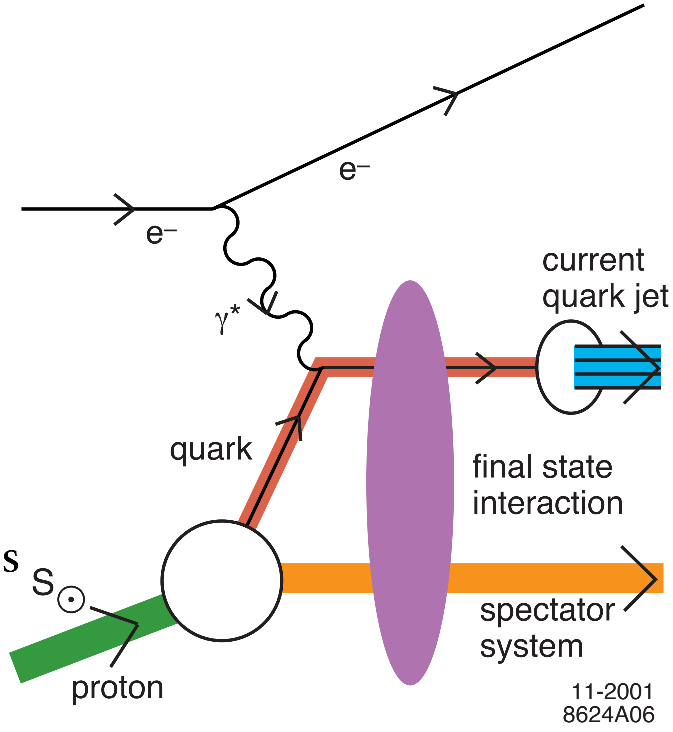
Analog of QED FSIs

Light-Front Wavefunction S and P- Waves

Final State Interactions not suppressed!

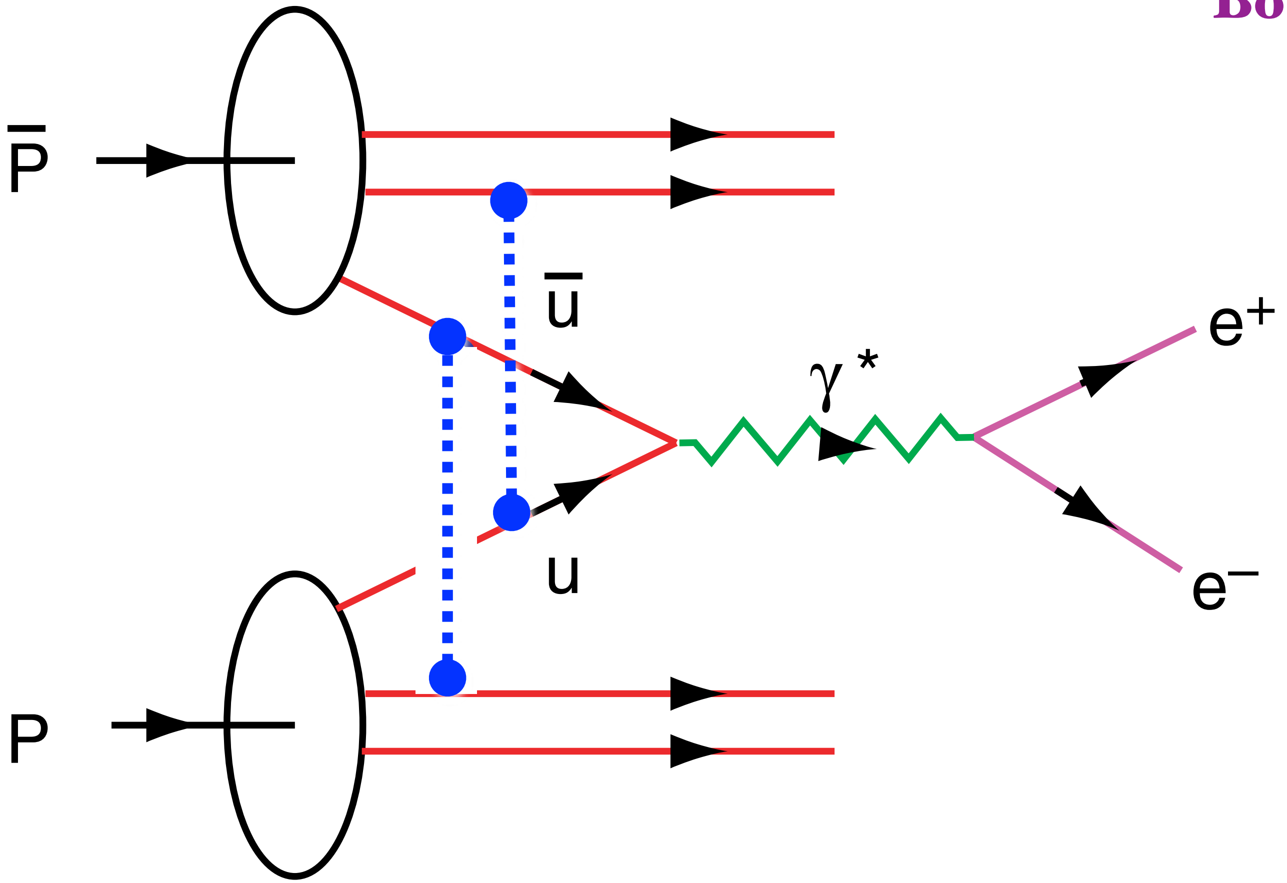
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$



- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

Boer, Hwang, sjb



DY $\cos 2\phi$ **correlation at leading twist from double ISI**

Product of Boer - Mulders Functions

$$h_1^\perp(x_1,\boldsymbol{p}_\perp^2)\times\bar{h}_1^\perp(x_2,\boldsymbol{k}_\perp^2)$$

Initial-State Interactions not suppressed!

Double Initial-State Interactions generate anomalous $\cos 2\phi$

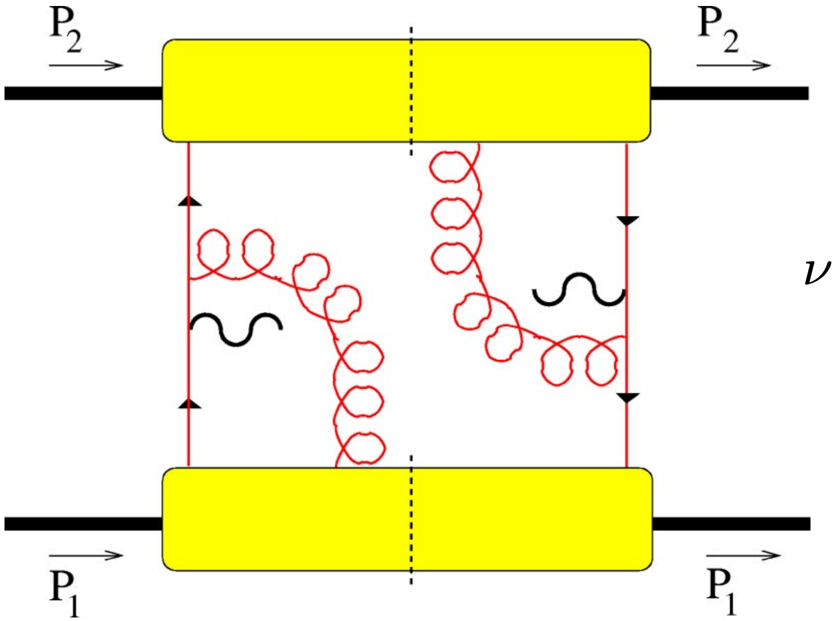
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

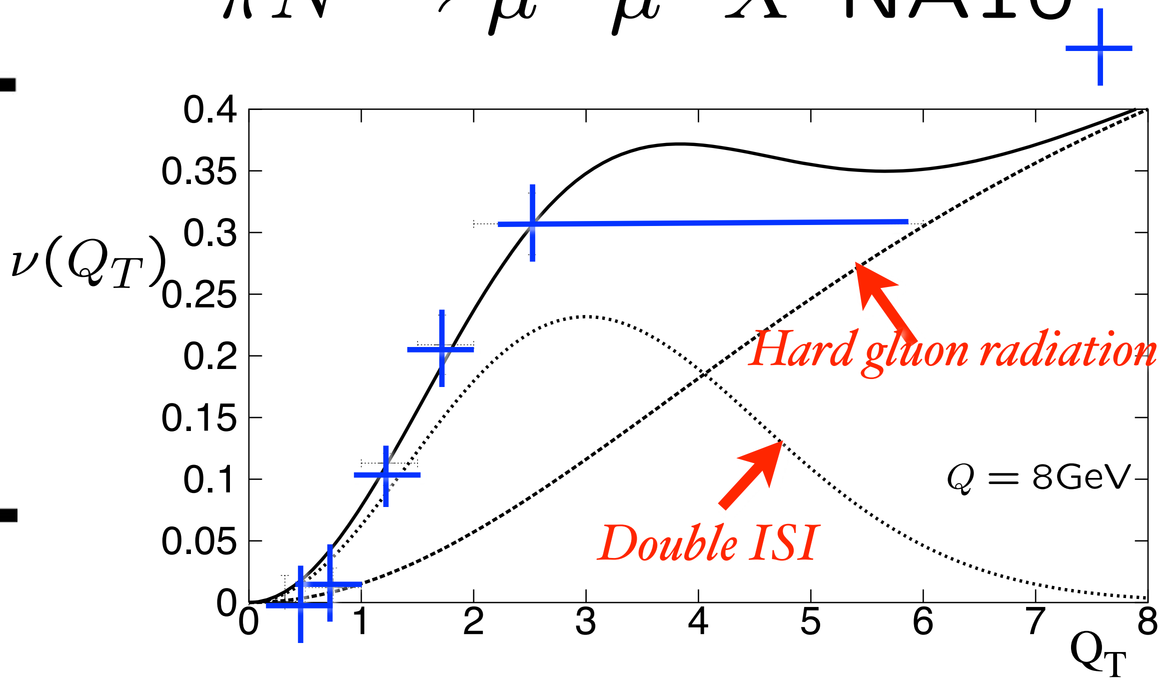
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$



Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



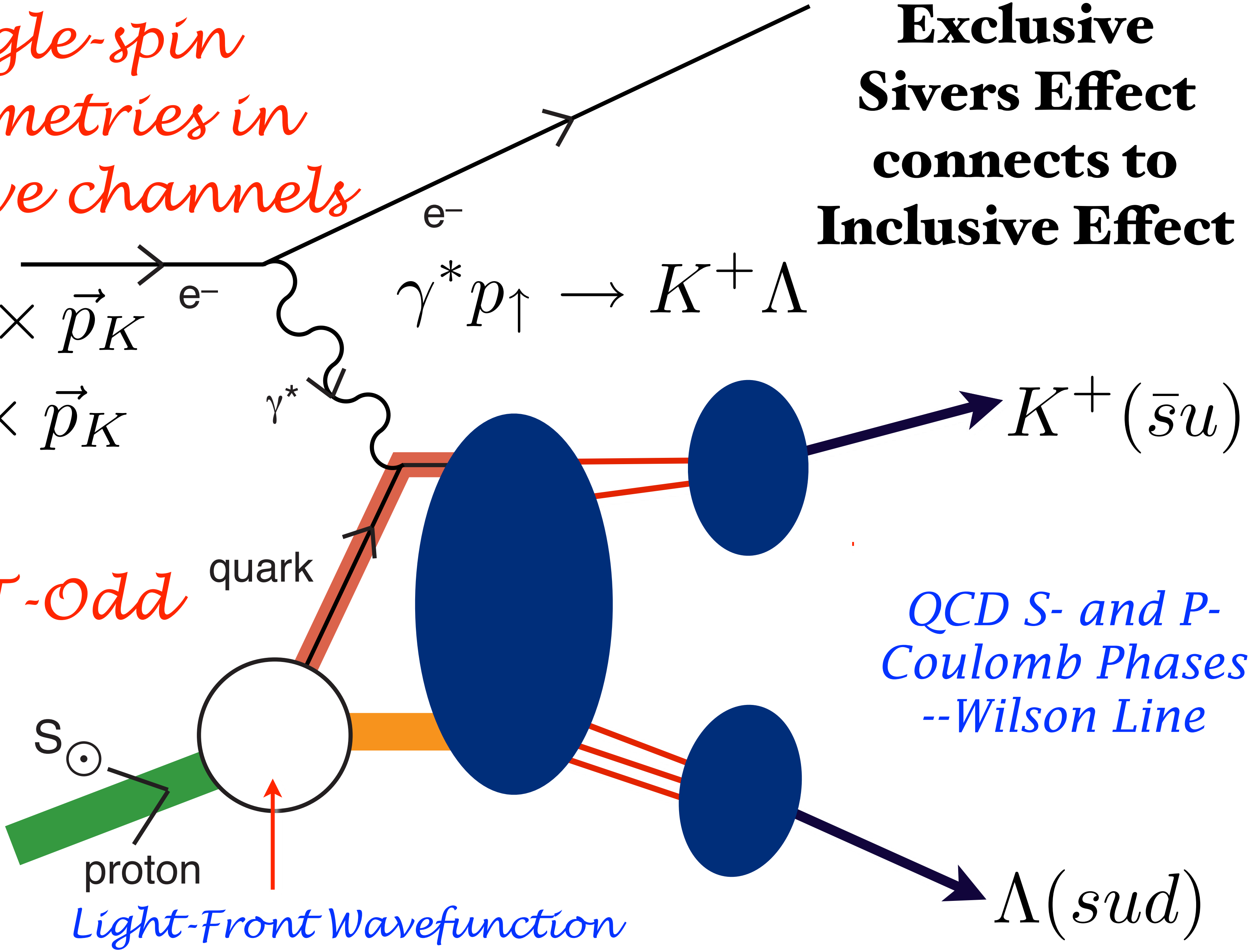
Model: Boer,

Single-spin asymmetries in exclusive channels

Exclusive Sivers Effect connects to Inclusive Effect

$$i\vec{S}_\Lambda \cdot \vec{q} \times \vec{p}_K$$

$$i\vec{S}_p \cdot \vec{q} \times \vec{p}_K$$



Pseudo-T-Odd

QCD S- and P- Coulomb Phases --Wilson Line

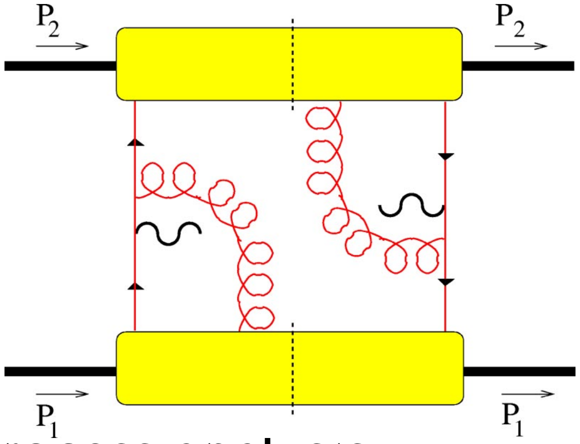
Light-Front Wavefunction S and P- Waves

JLab Expt

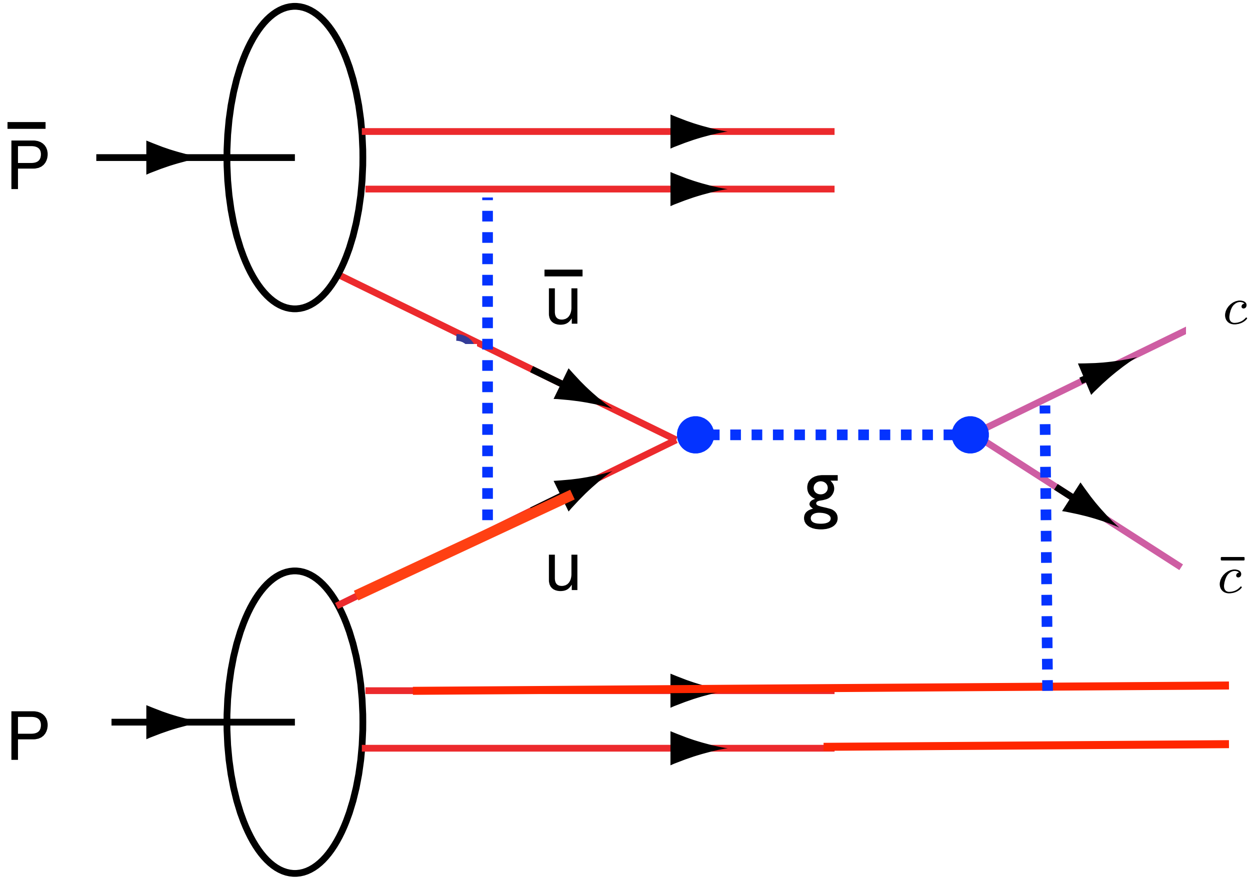
Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation

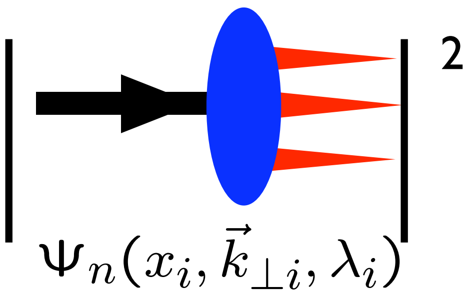


- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

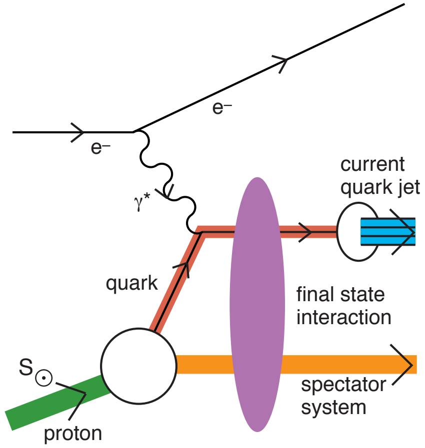


Problem for factorization when both ISI and FSI occur!

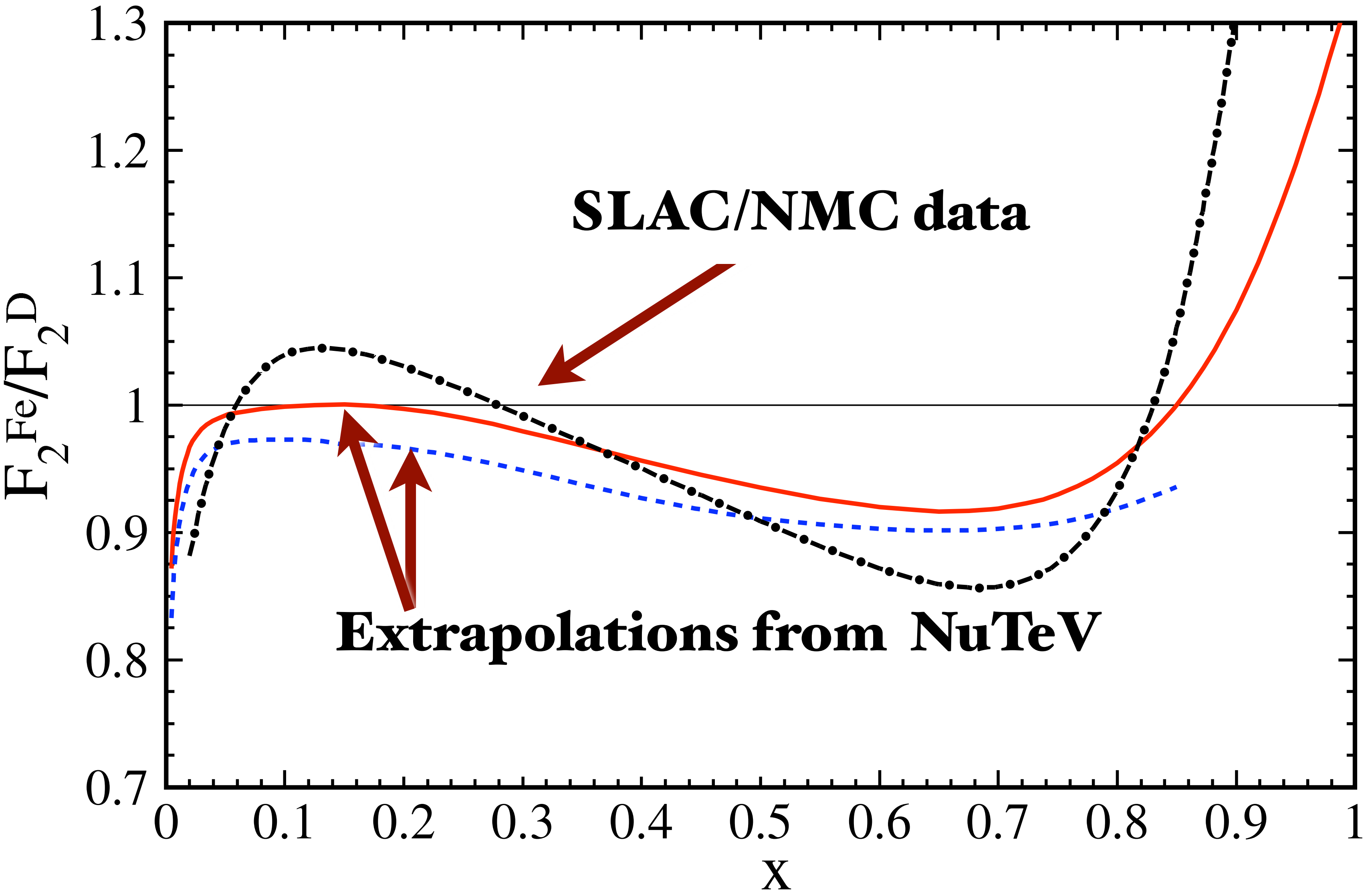
Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS



$$\left|\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$



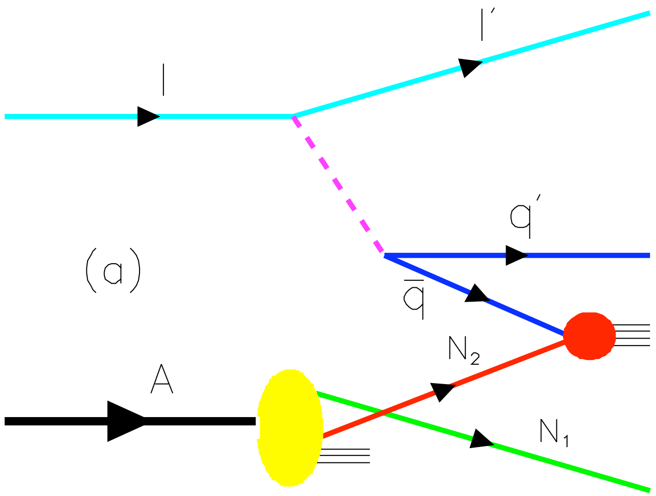
$Q^2 = 5\ \mathrm{GeV}^2$



Scheinbein, Yu, Keppel, Morfin, Olness, Owens

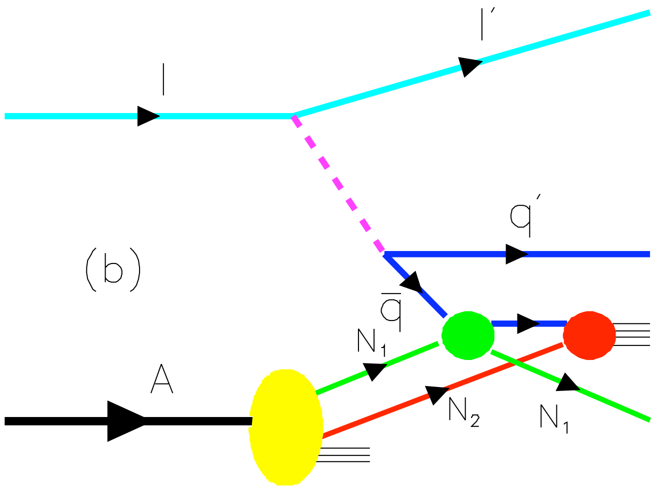
Novel World of Hadron Physics

Stan Brodsky



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.



If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

→ Shadowing of the DIS nuclear structure functions.

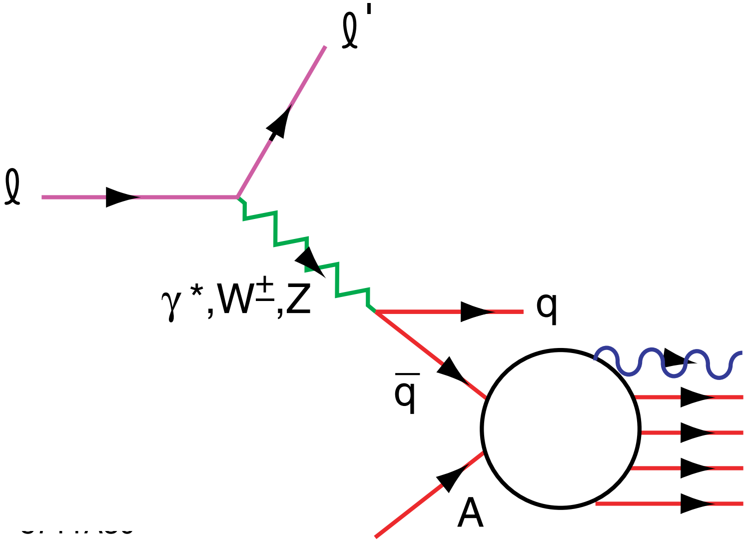
Diffraction via Reggeon gives constructive interference!

Anti-shadowing

Origin of Regge Behavior of Deep Inelastic Structure Functions

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$

Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$



Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

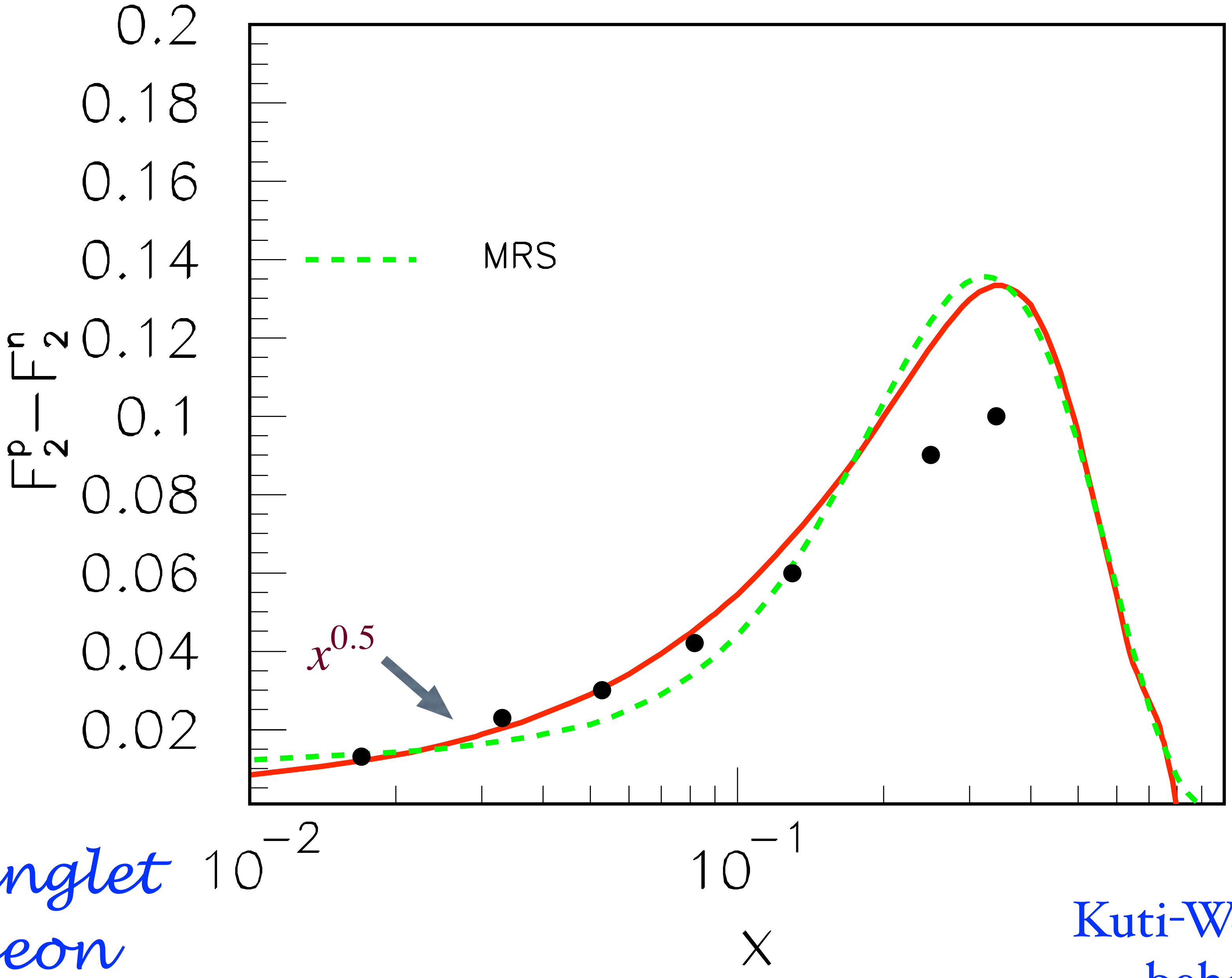
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x}_{bj}$ at small x_{bj}.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Schmidt, Yang, Lu, sjb



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i) \times i = \frac{1}{\sqrt{2}}(i+1)$$

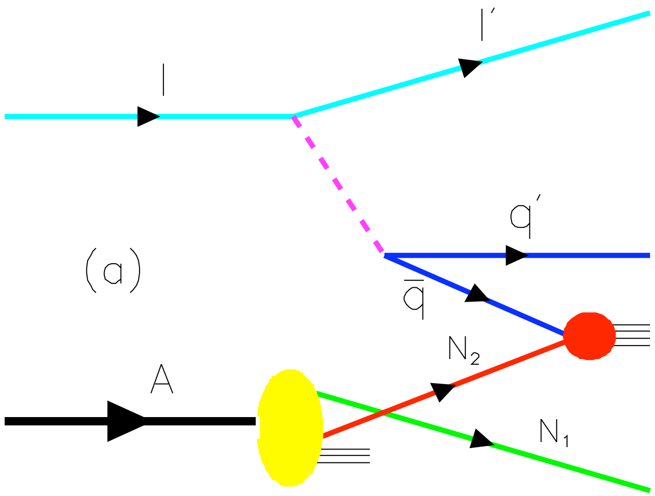
Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^{\pm}$

Critical test: Tagged Drell-Yan



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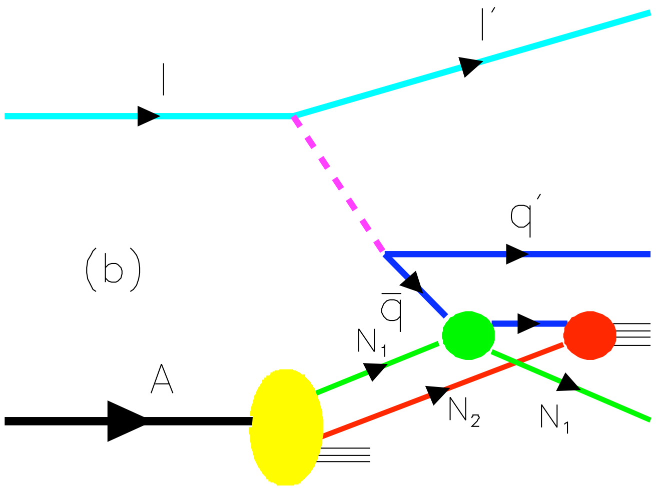

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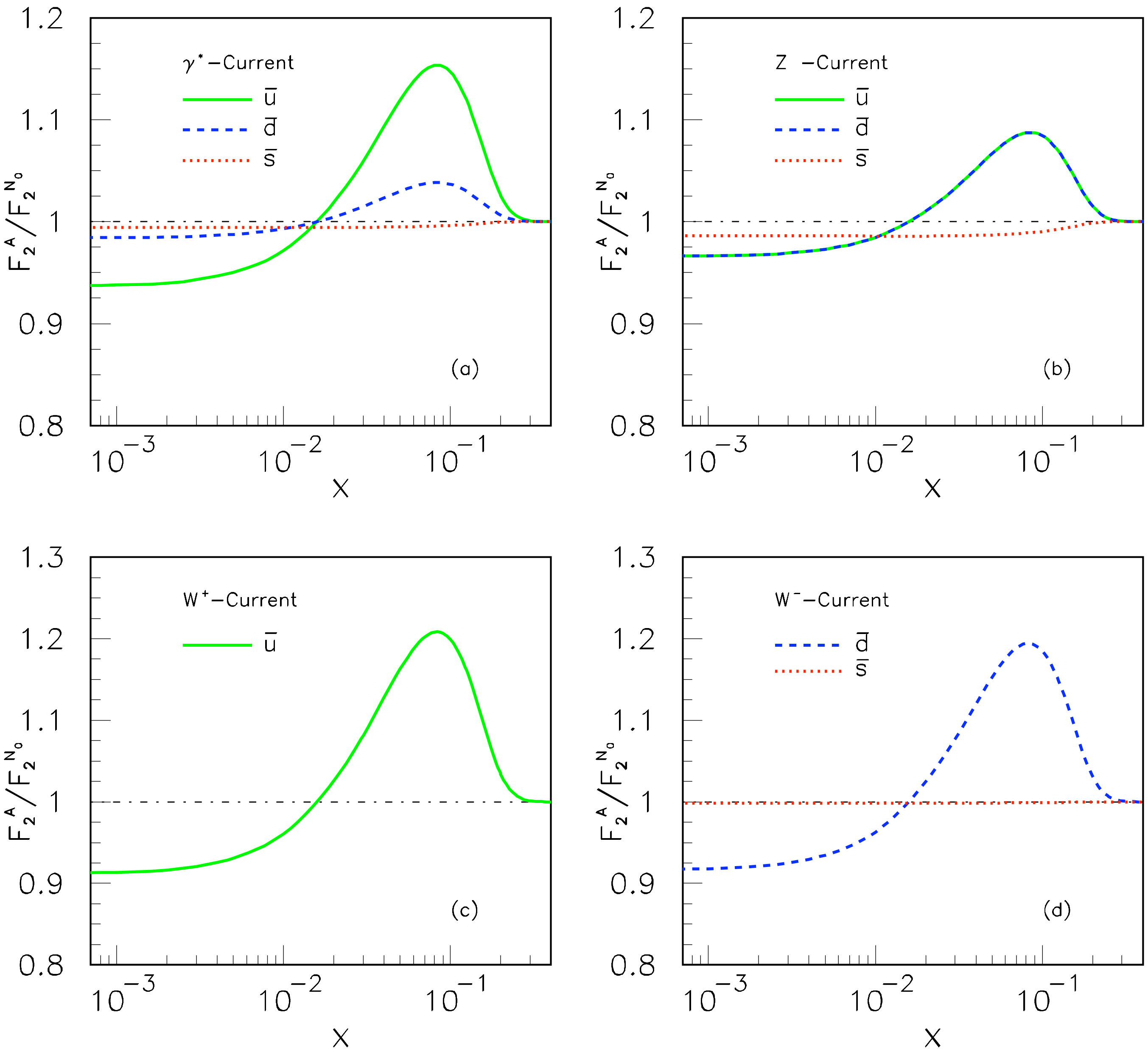


Schmidt, Yang; sjb

Nuclear Antishadowing not universal !

Test at JLab — Flavor tagged Structure Functions

QED atoms: positronium and muonium

$$H_{QED}$$

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

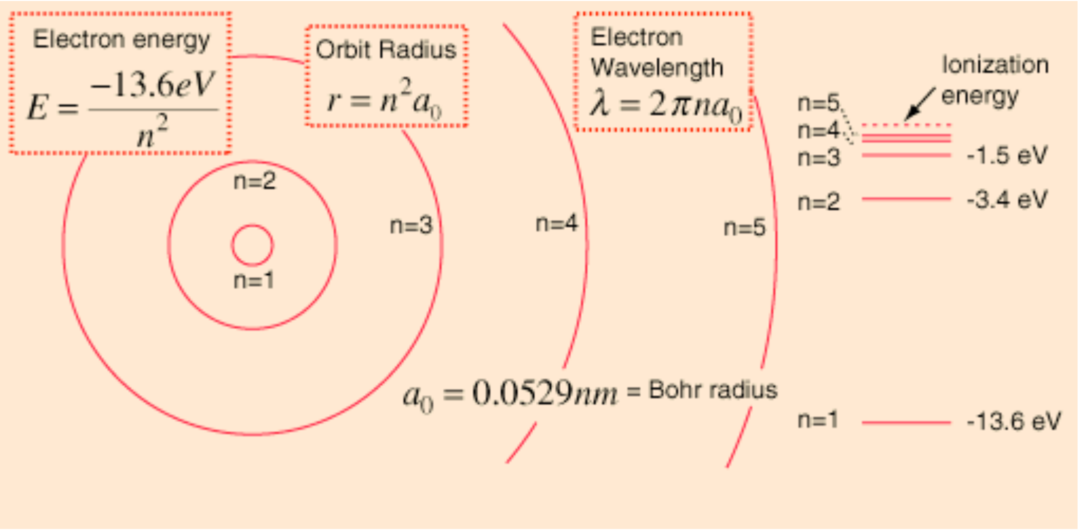
Semiclassical first approximation to QED

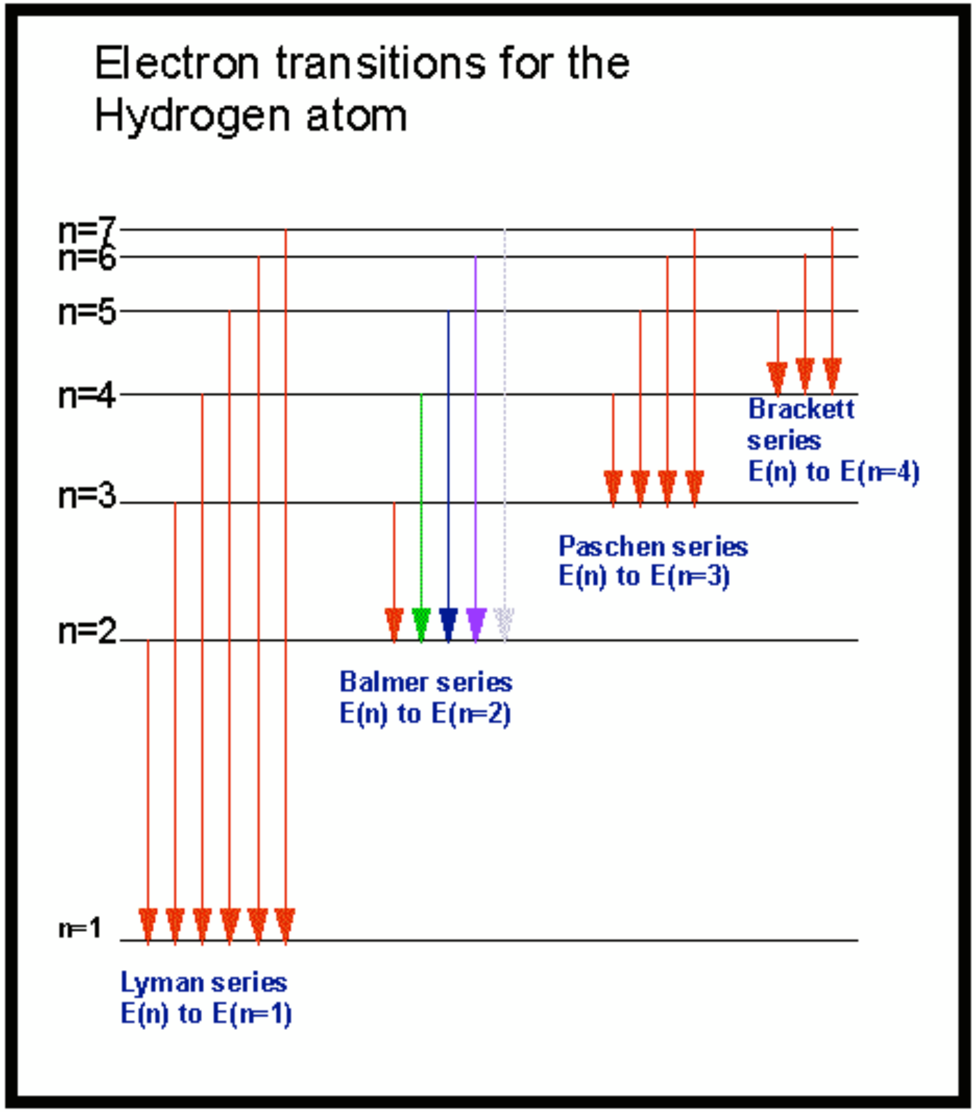
Coulomb potential

Bohr Spectrum

Schrödinger Eq.

Bohr Atom




Electron transitions for the Hydrogen atom

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What sets the QCD mass scale?
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Structure Functions,Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates

$$\boxed{H^{LF}_{QCD}}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\boxed{[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)}$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis ζ, ϕ

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD
potential

Semiclassical first approximation to QCD

Light-Front Schrödinger Equation

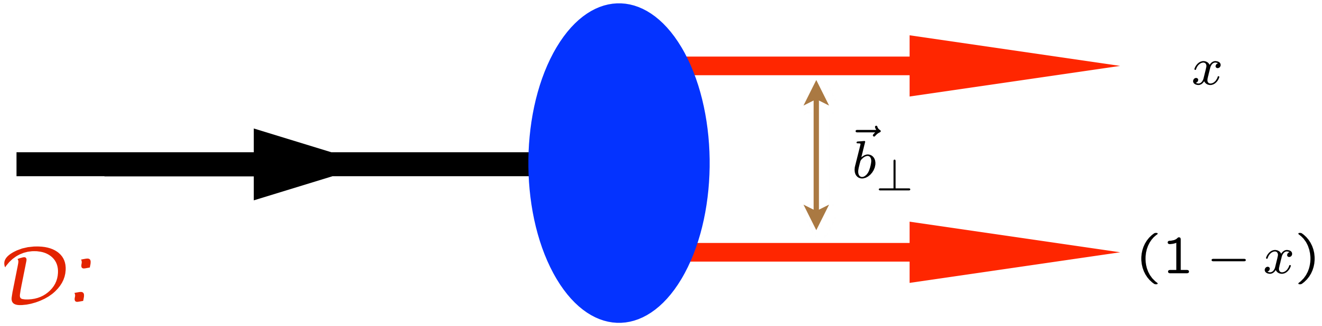
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

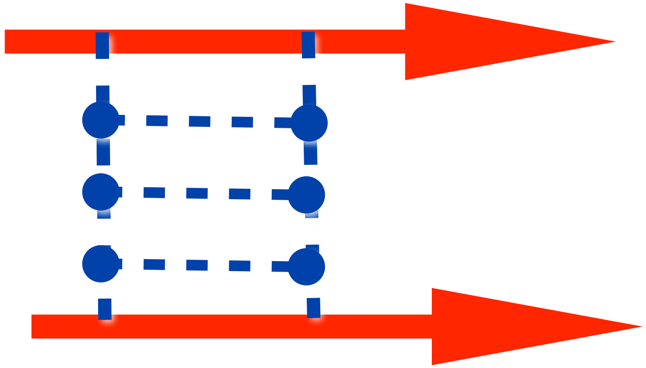
$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



AdS/QCD:

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

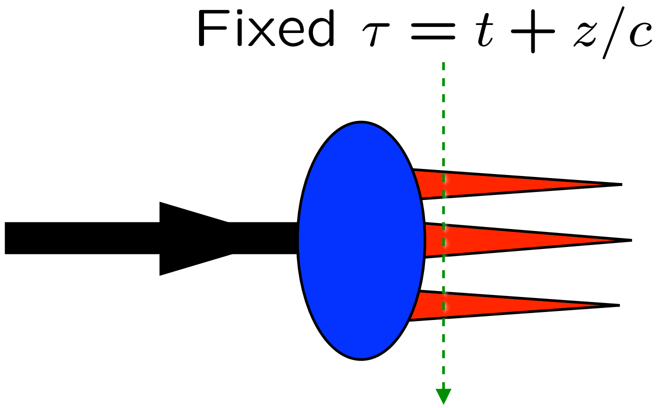


U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

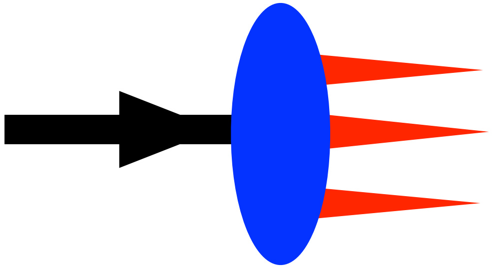
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

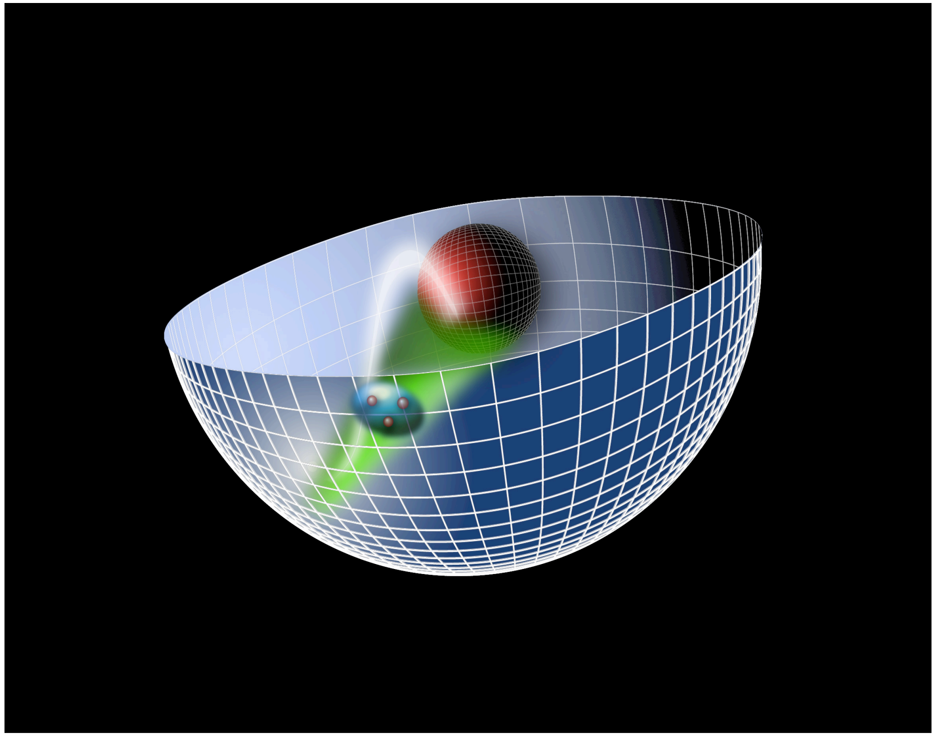
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct
a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc

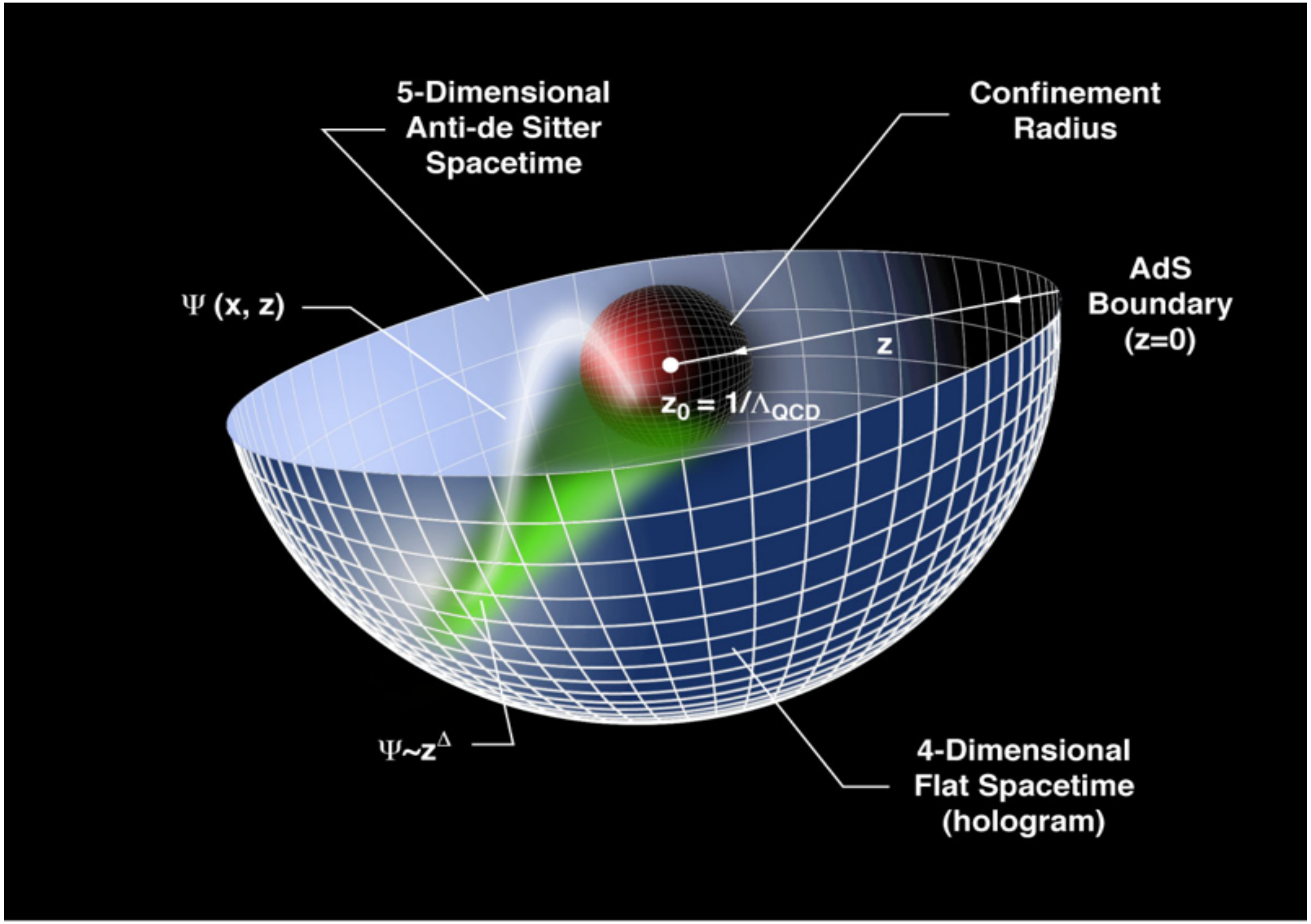


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



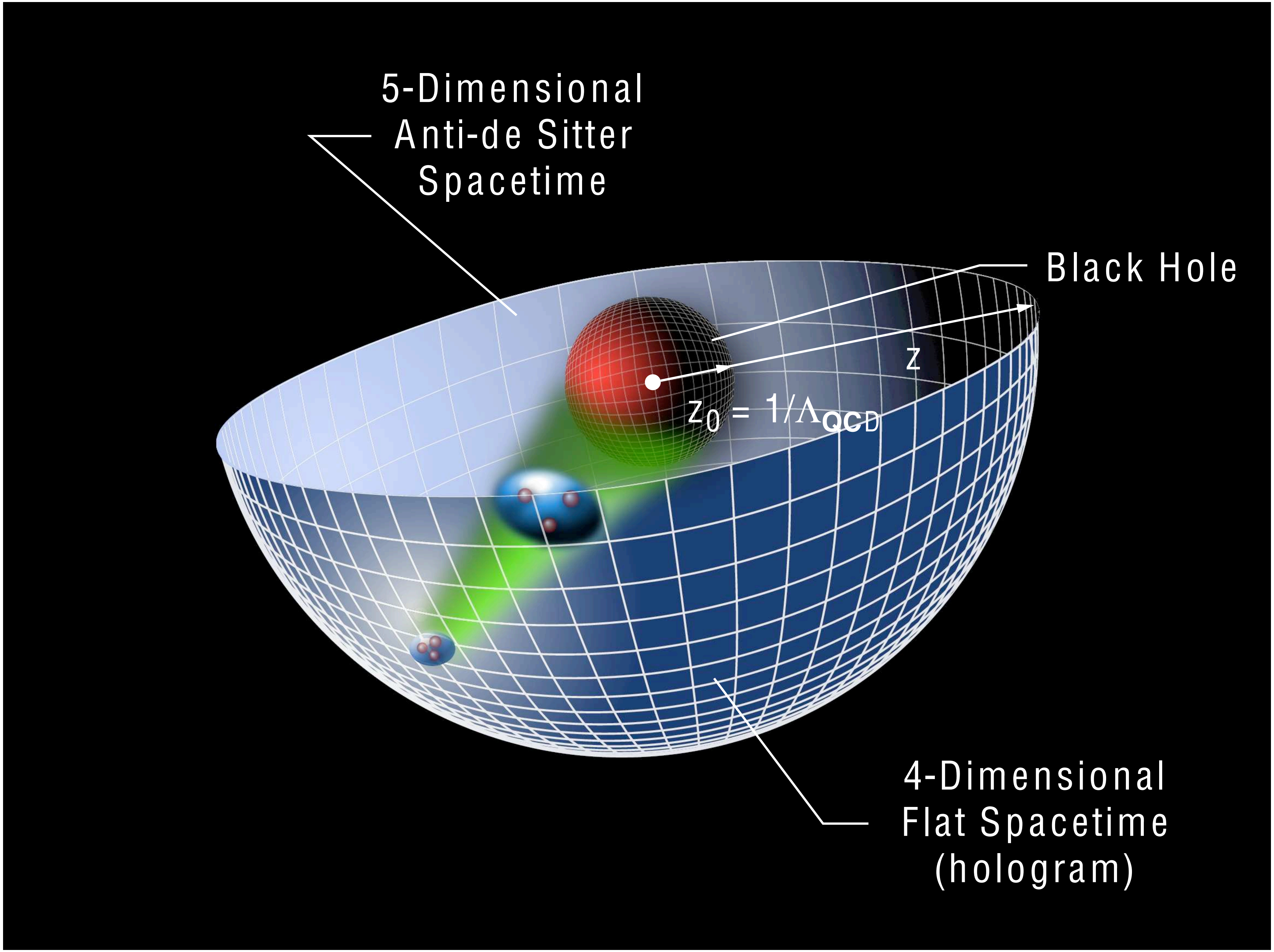
in collaboration with
Guy de Teramond

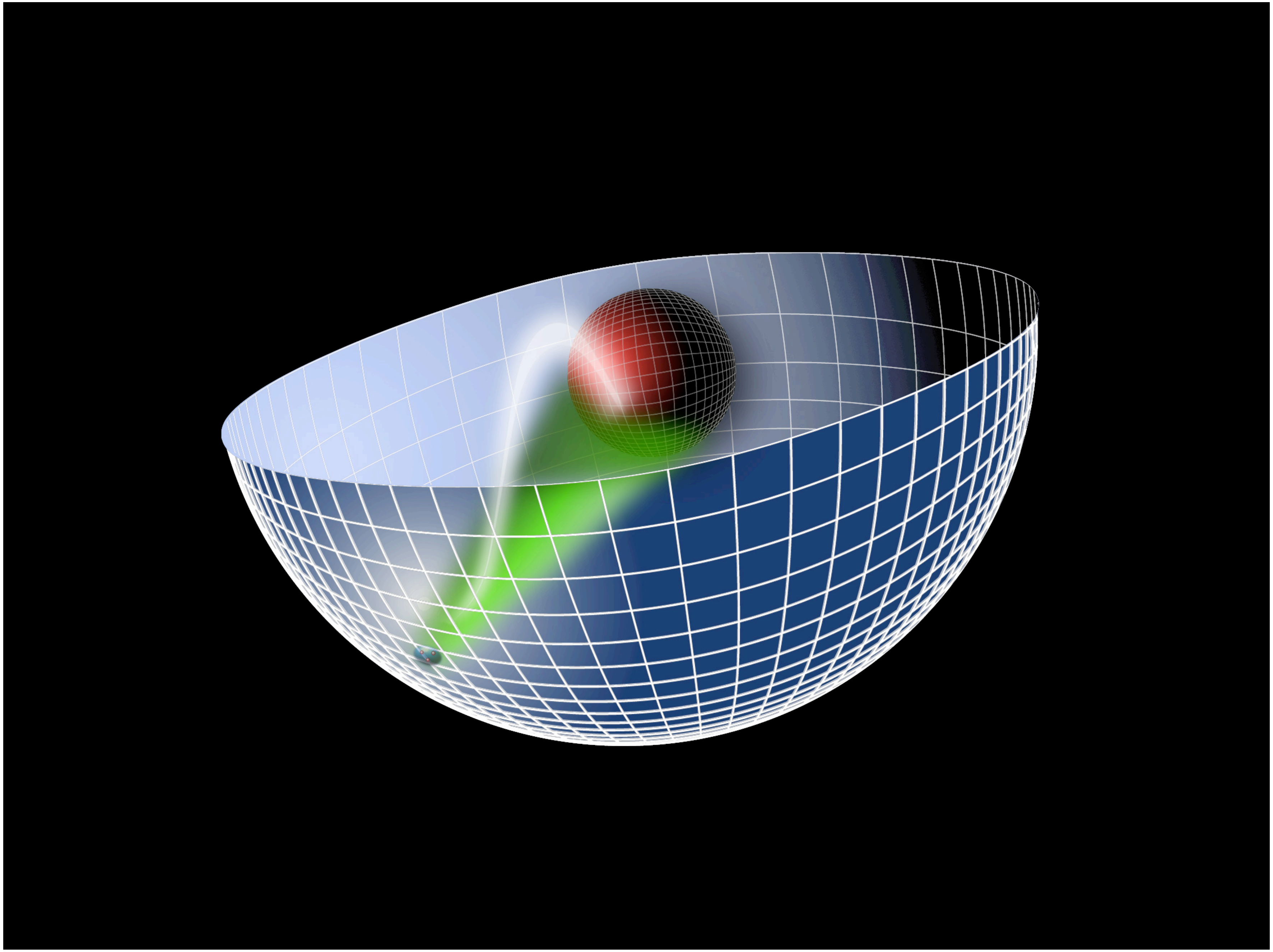
Applications of AdS/CFT to QCD

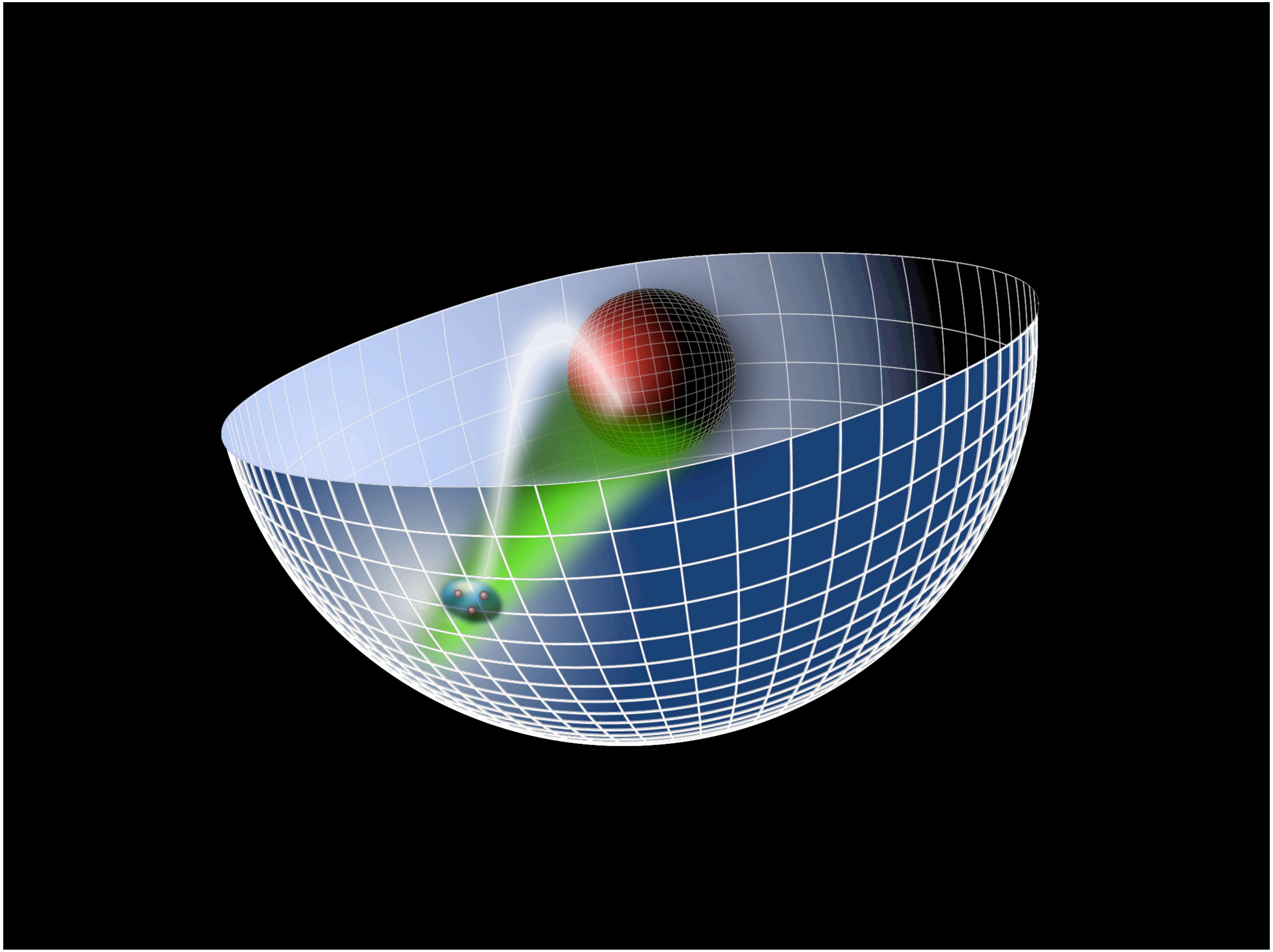


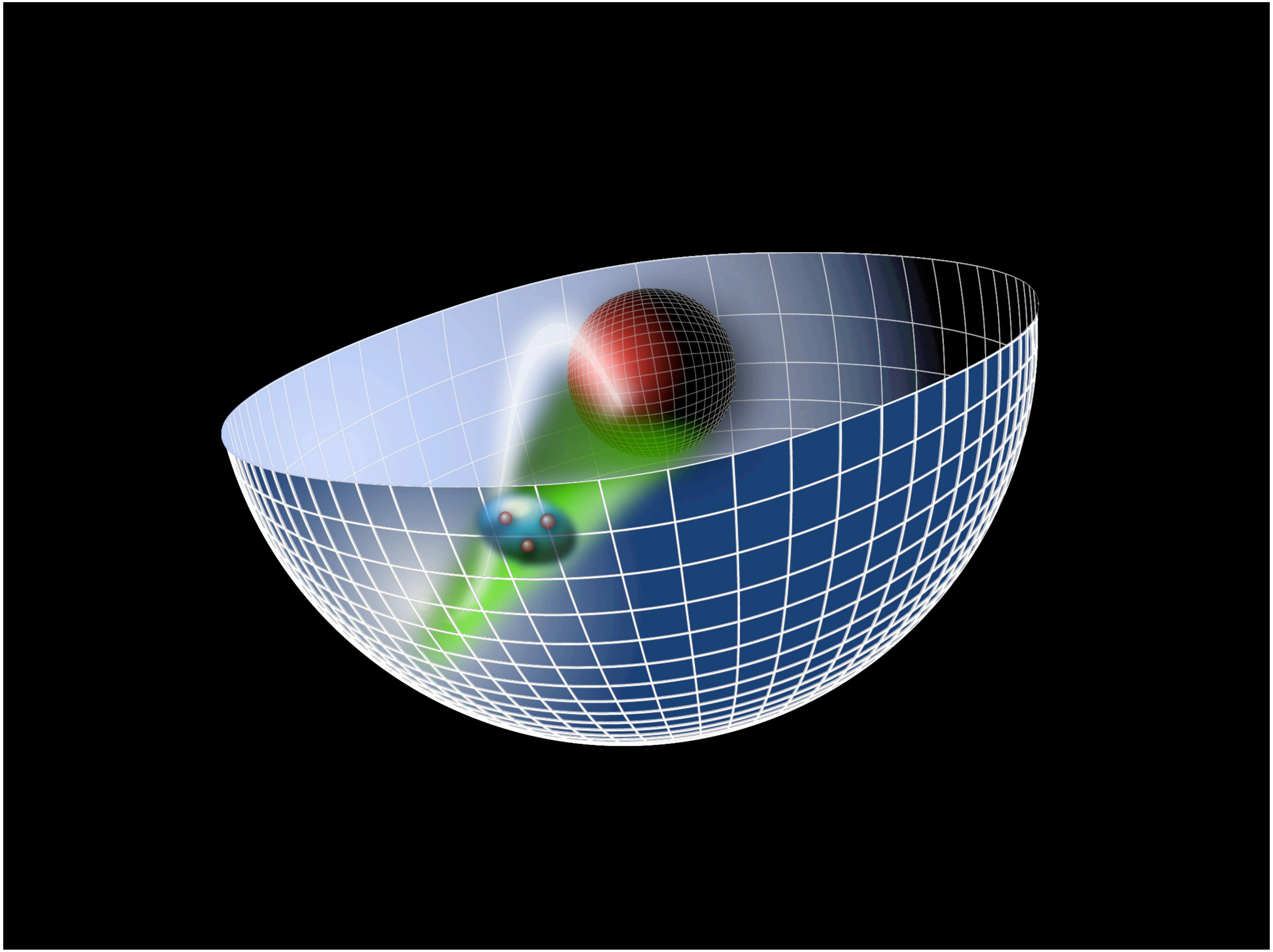
Changes in physical length scale mapped to evolution in the 5th dimension z

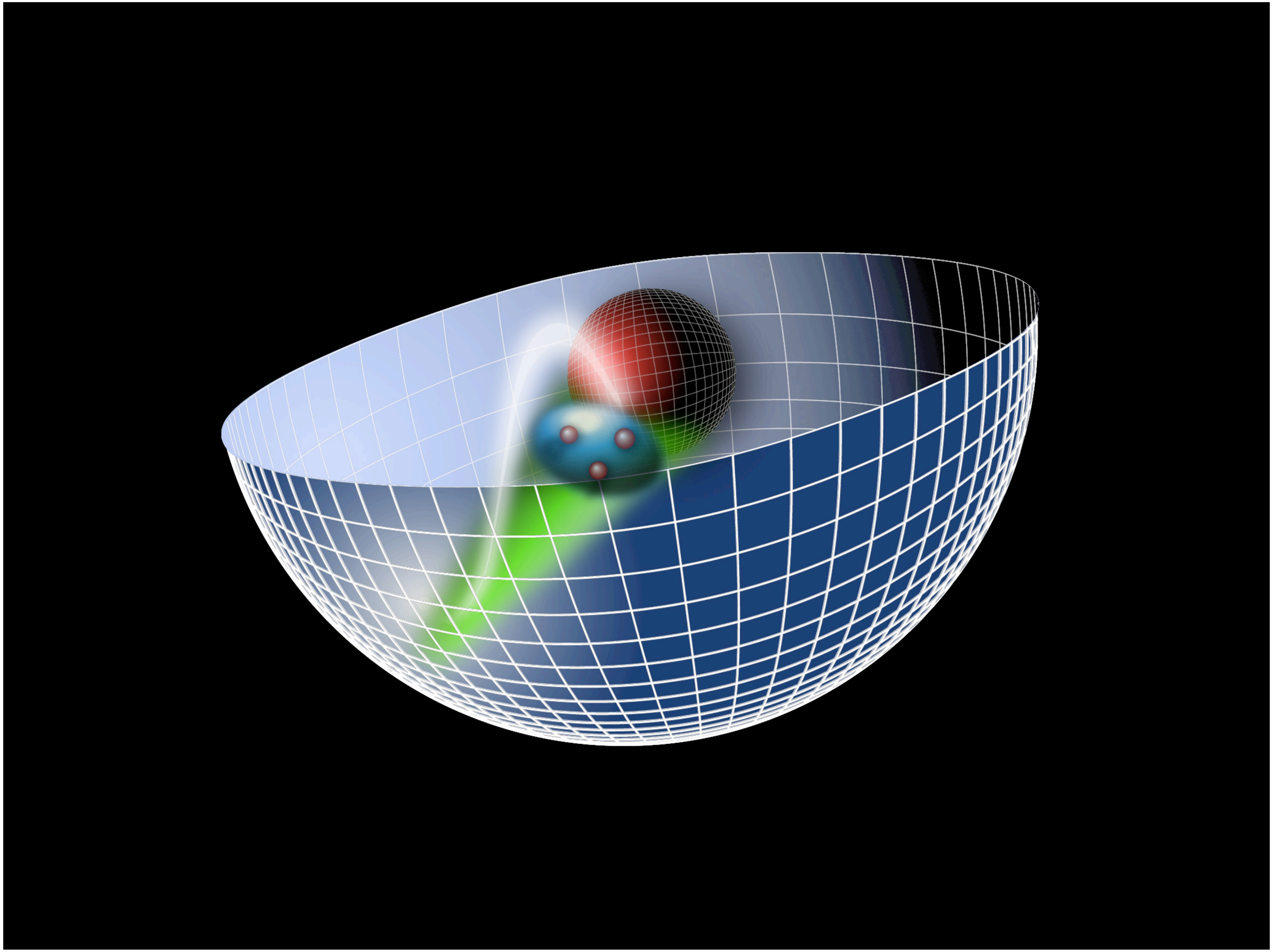
in collaboration with Guy de Teramond


5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

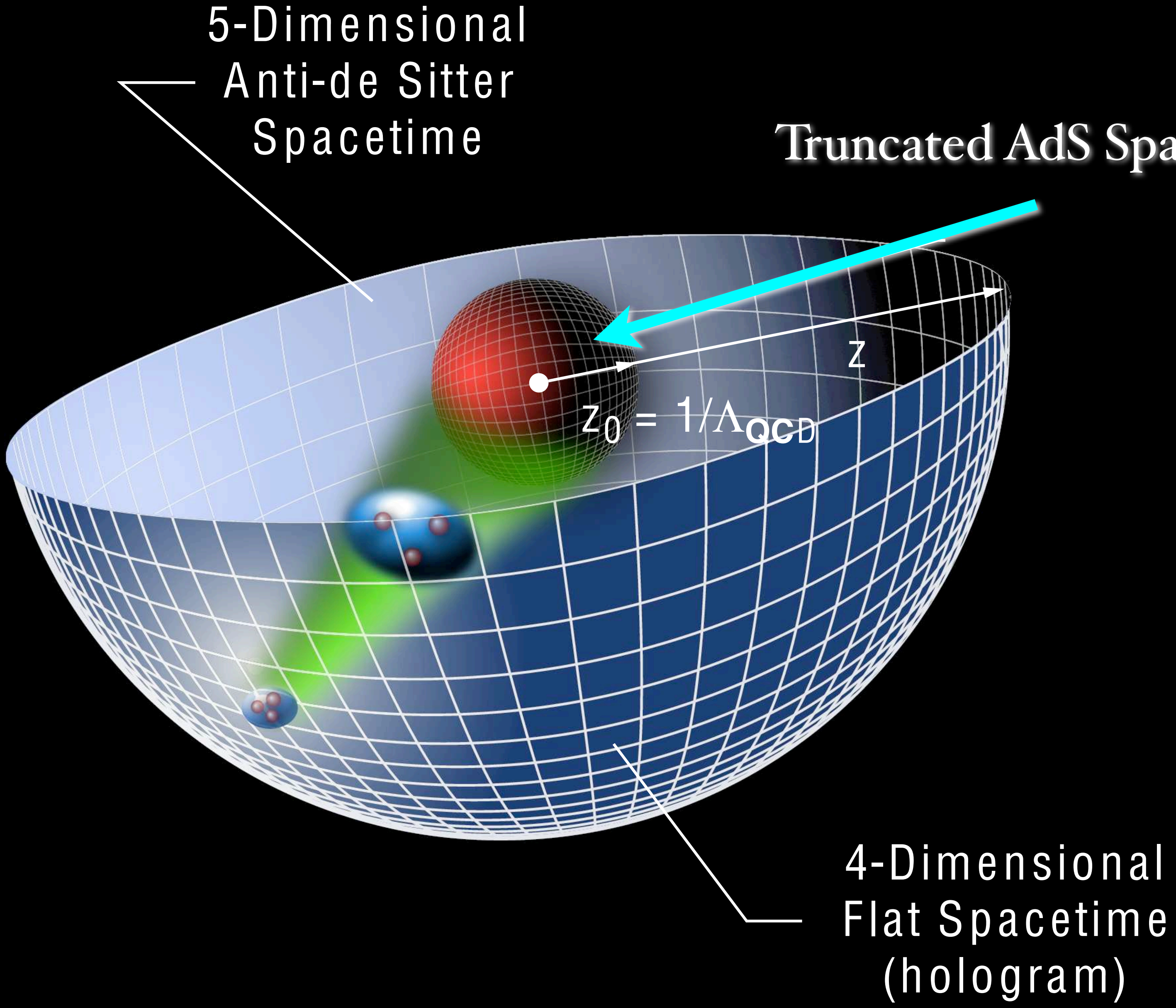


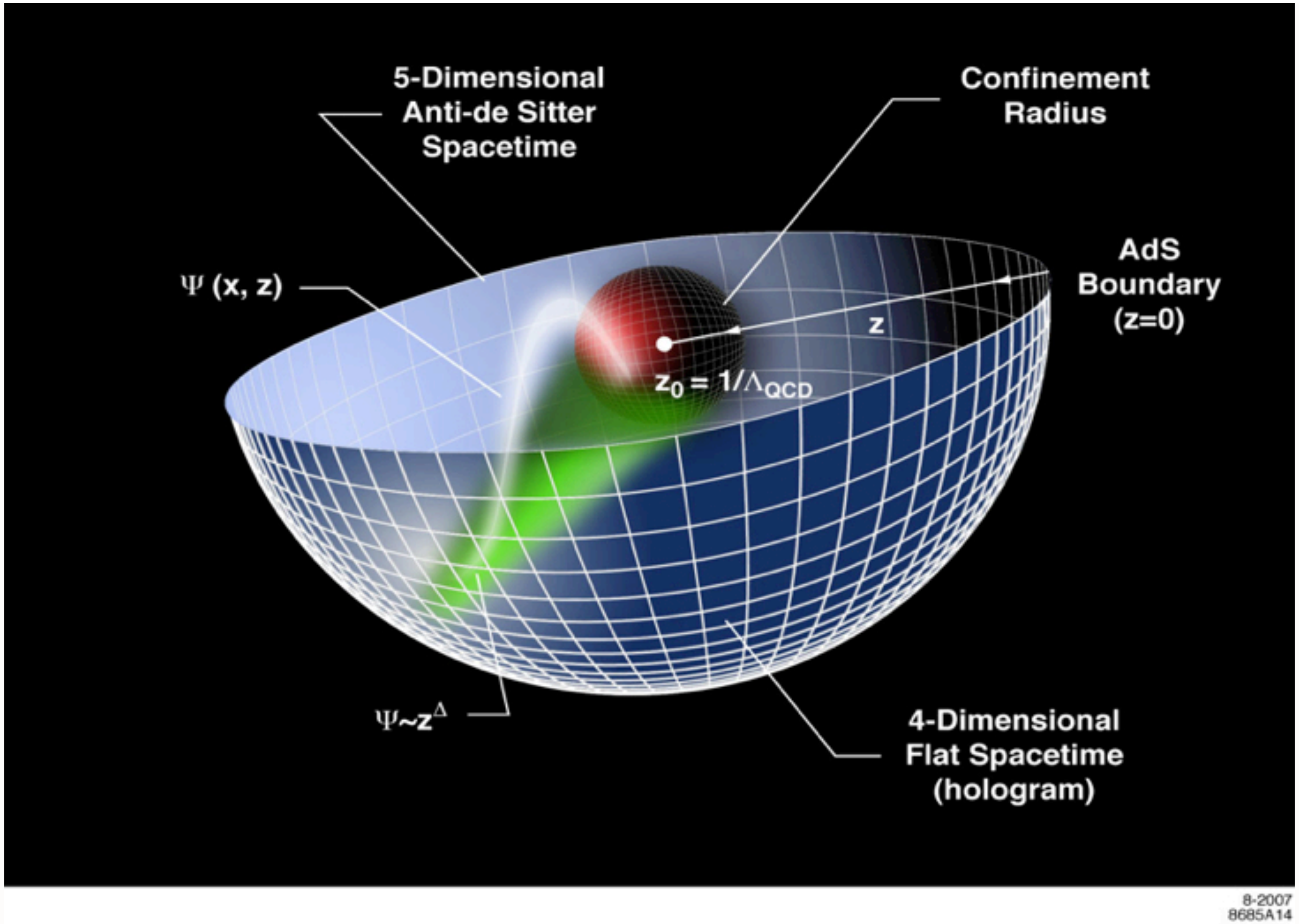






Truncated AdS Space





Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu}x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS$_5$ as template for conformal theory**

Introduce “Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

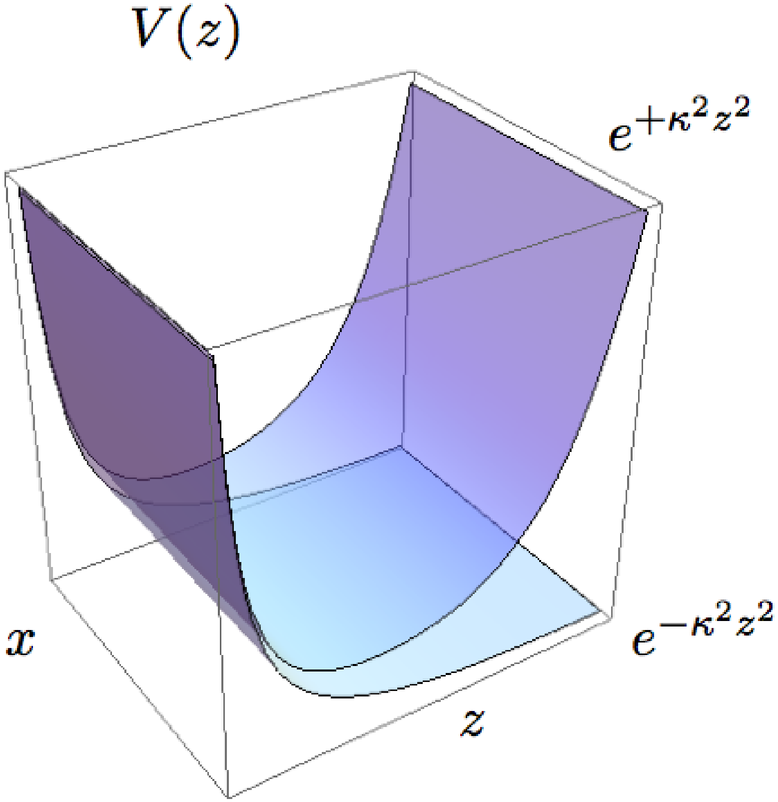
$$ds^2 = \frac{R^2}{z^2}\boxed{e^{\varphi(z)}}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS$_5$

- Gravitational potential energy for object of mass m

$$V = mc^2\sqrt{g_{00}} = mc^2 R\,\frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z\rangle \sim 1/\kappa$



Klebanov and Maldacena

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z}}$$

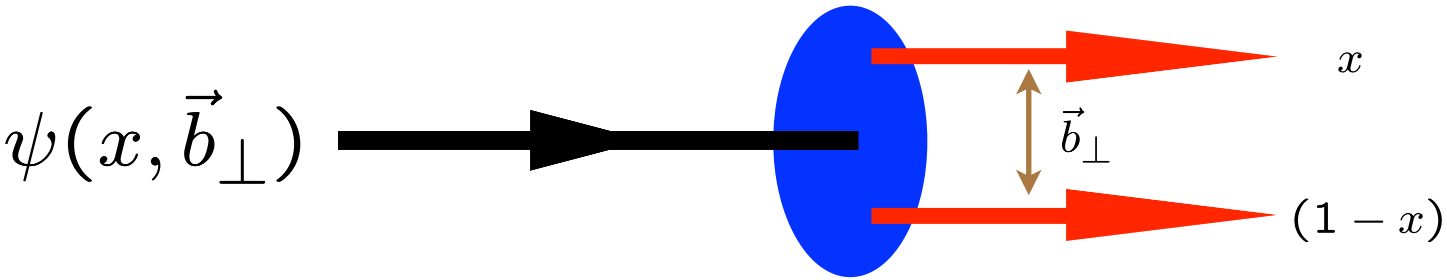
Positive-sign dilaton

- de Teramond, sjb

LF(3+1) AdS$_5$

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

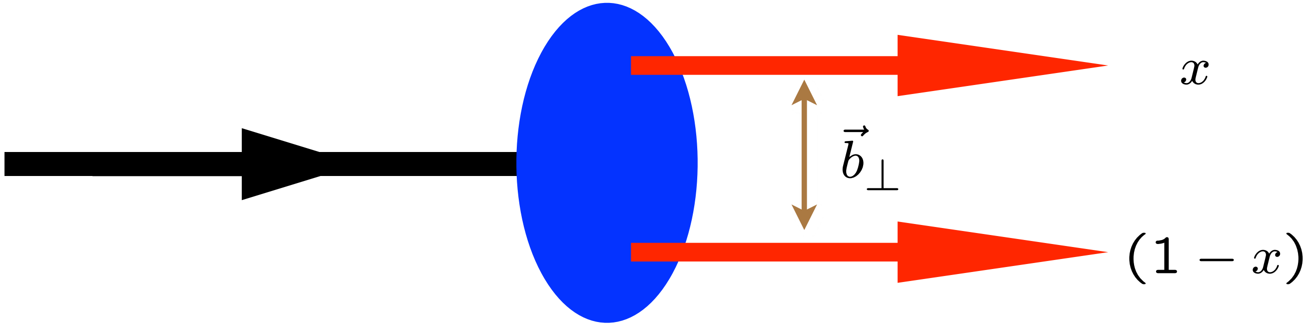
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall
confining potential:

de Teramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

- Substituting in the AdS scalar wave equation for Φ

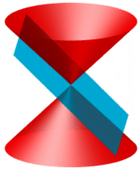
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

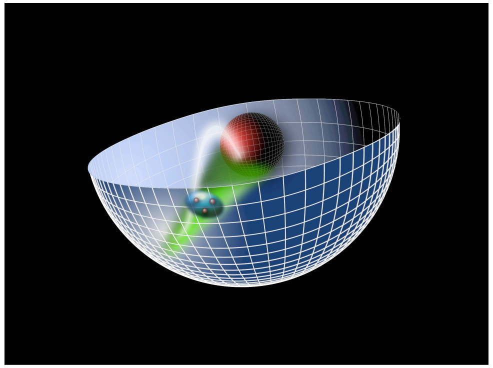


with $(\mu R)^2 = -(2 - J)^2 + L^2$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry
of the action

Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to Schrödinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

Light-Front Schrödinger Equation

Light-Front Schrödinger Equation

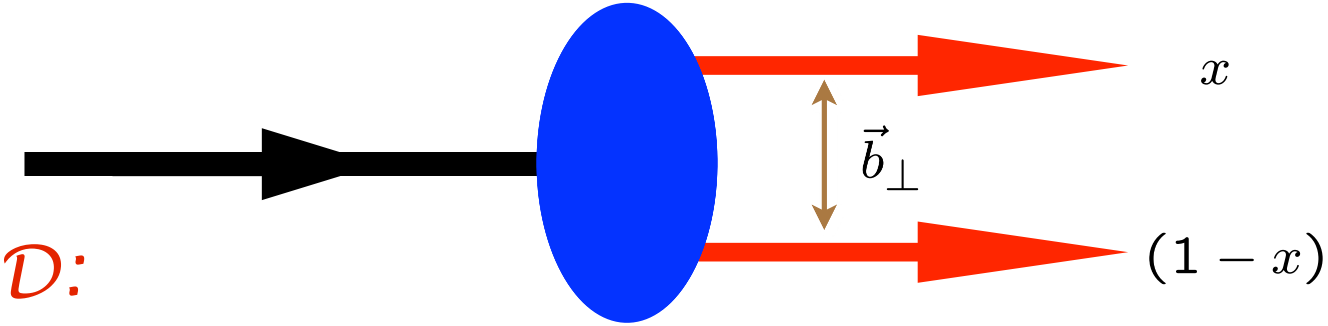
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Novel World of Hadron Physics

Stan Brodsky

Soft Wall Model

Quark separation increases with L

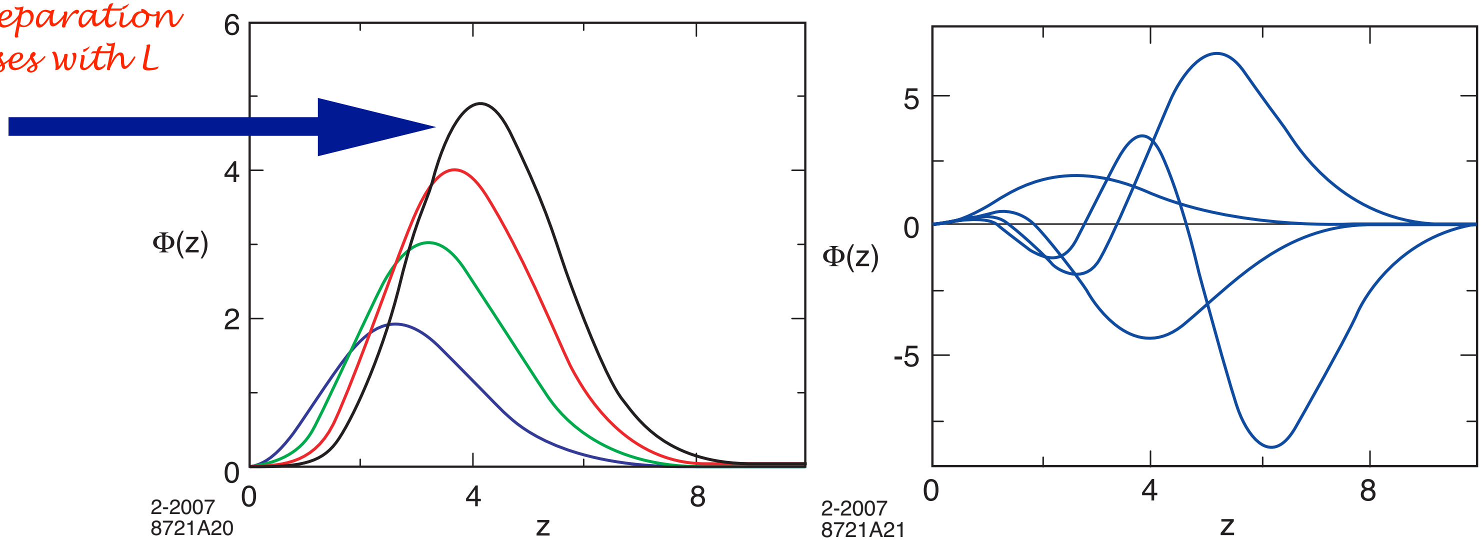
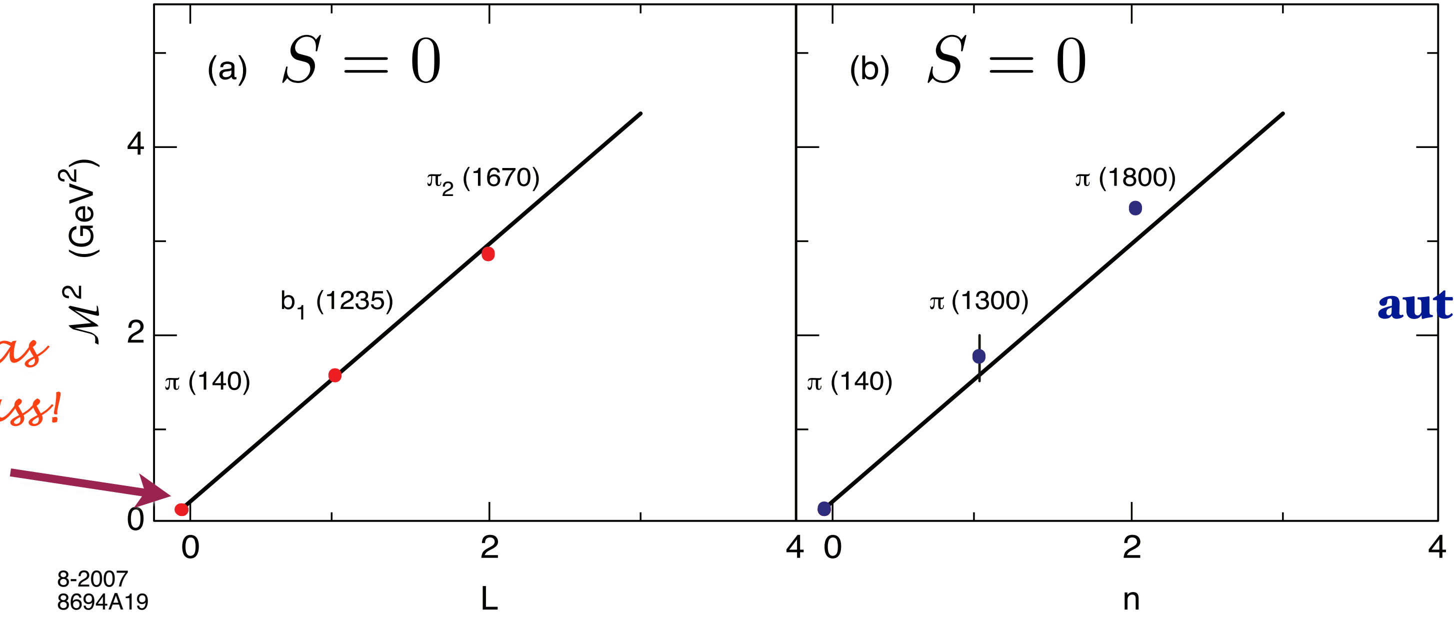


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!

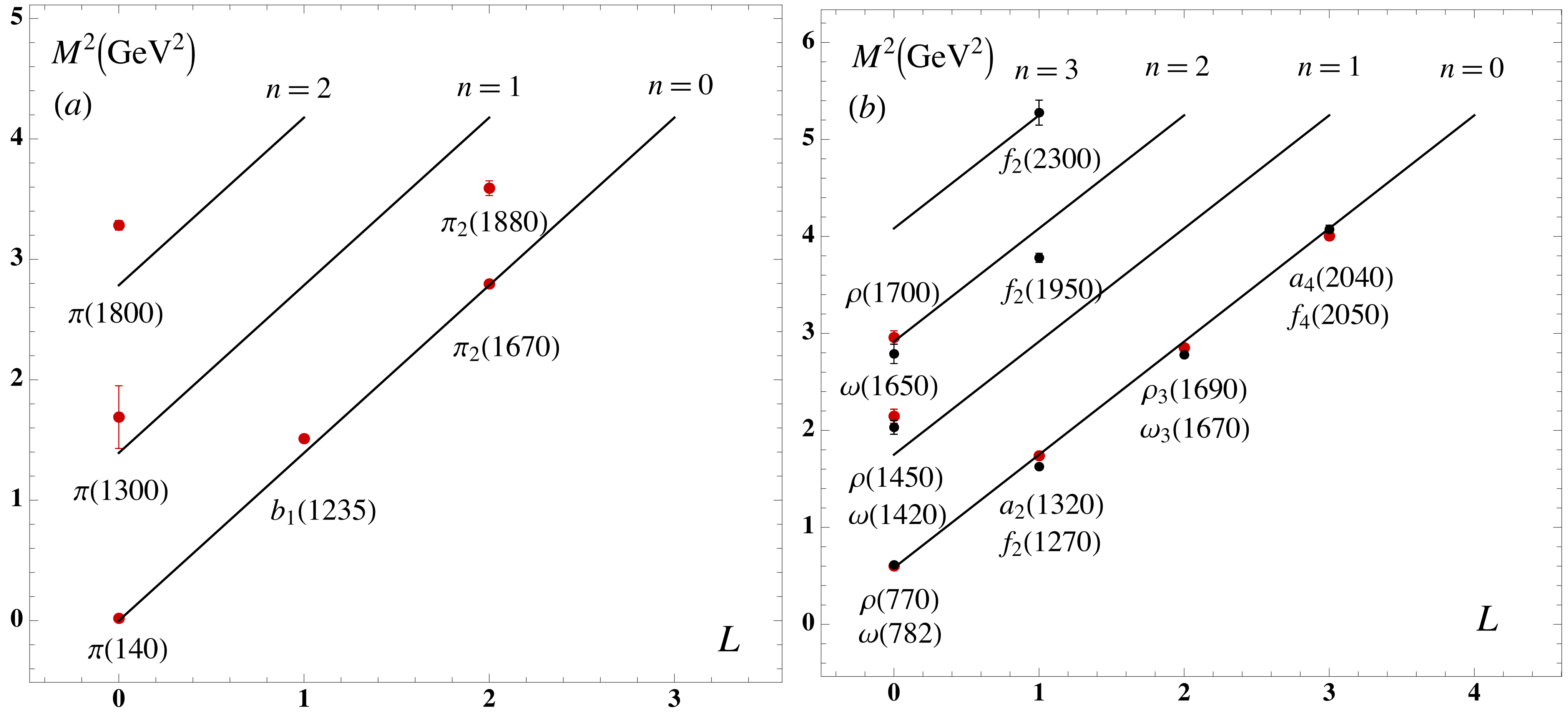


Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.



$$M^2(n, L, J) = 4\kappa^2(n + L/2 + J/2)$$

Novel World of Hadron Physics

Stan Brodsky

Bosonic Modes and Meson Spectrum

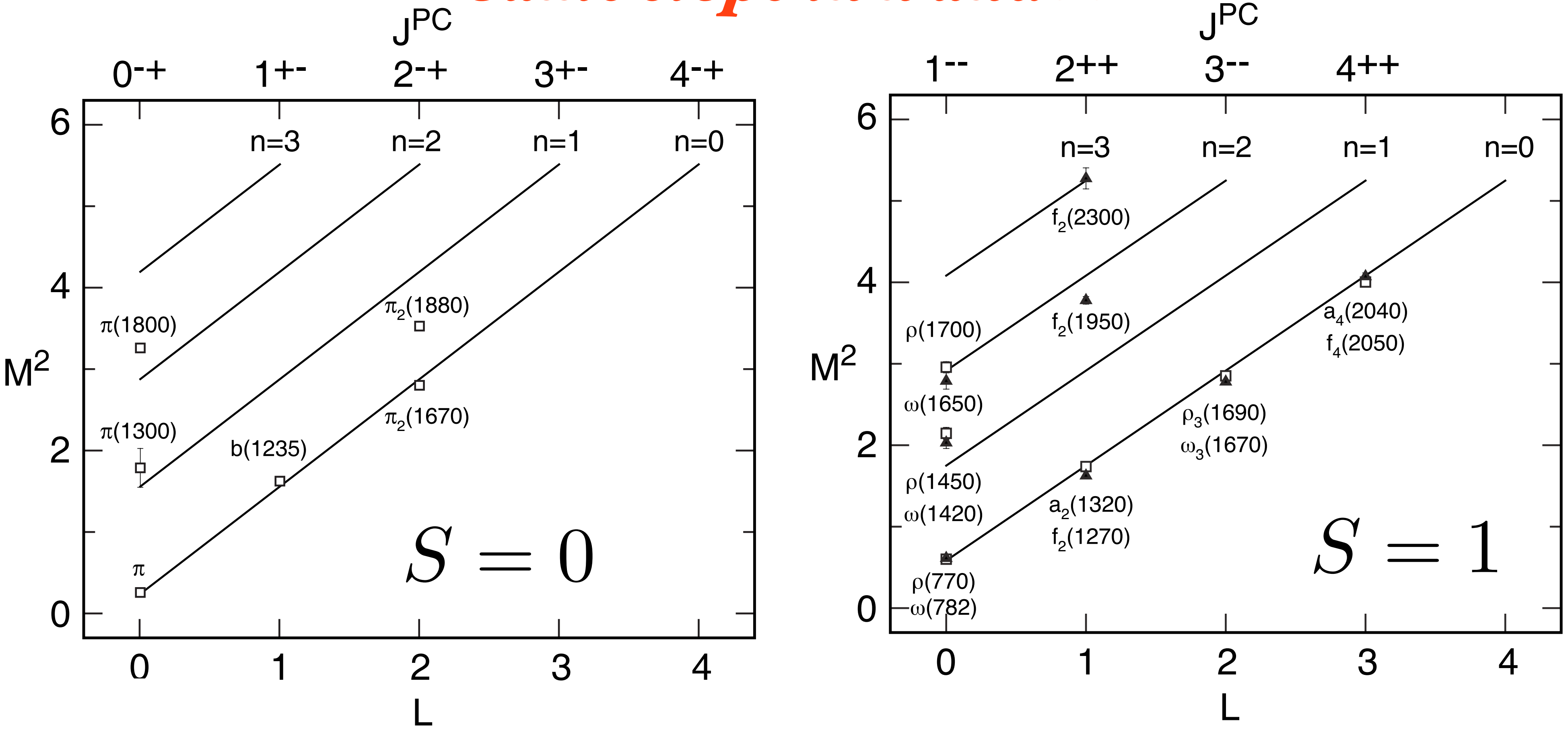
$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \to 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L



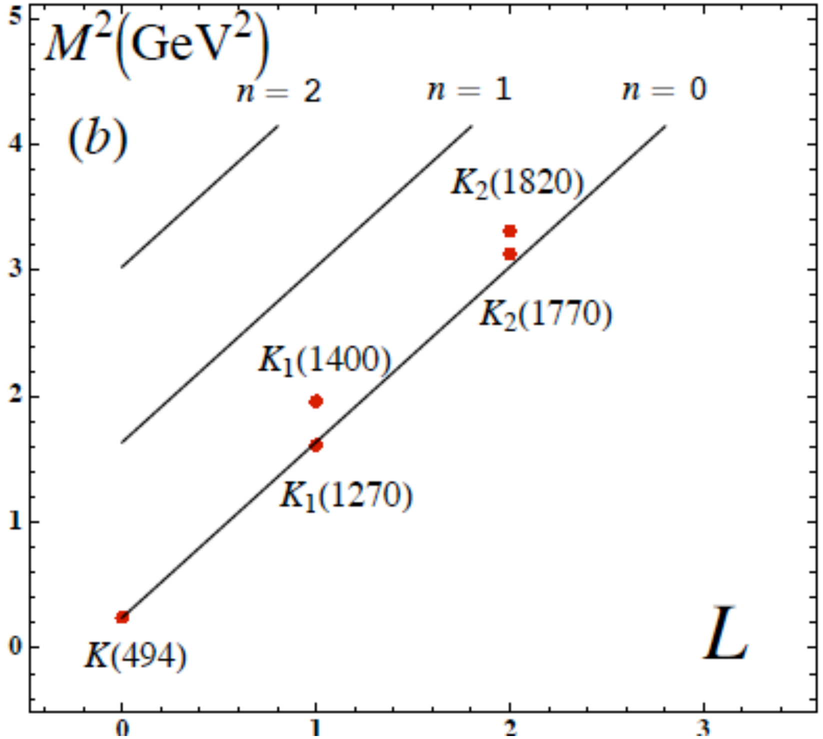
Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I=1$ ρ-meson and $I=0$ ω-meson families ($\kappa = 0.54$ GeV)

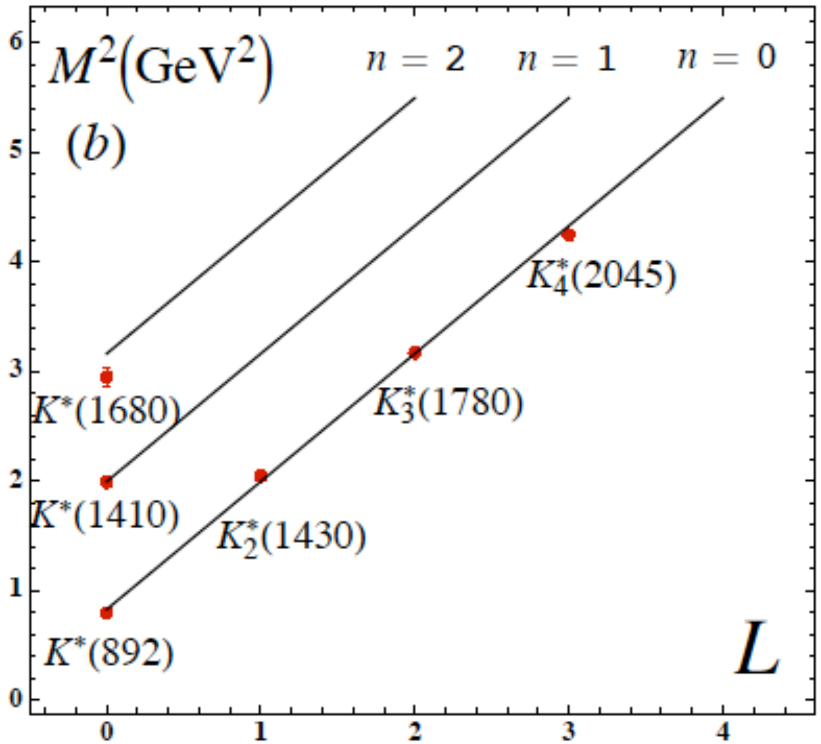
Balmer series of QCD

Kaon Spectrum

de Tèramond, Dosch, sjb

$$\mathcal{M}^2 = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$





$$\delta M^2 = \sum_i < \frac{m_i^2}{x_i} >$$

Weisberger

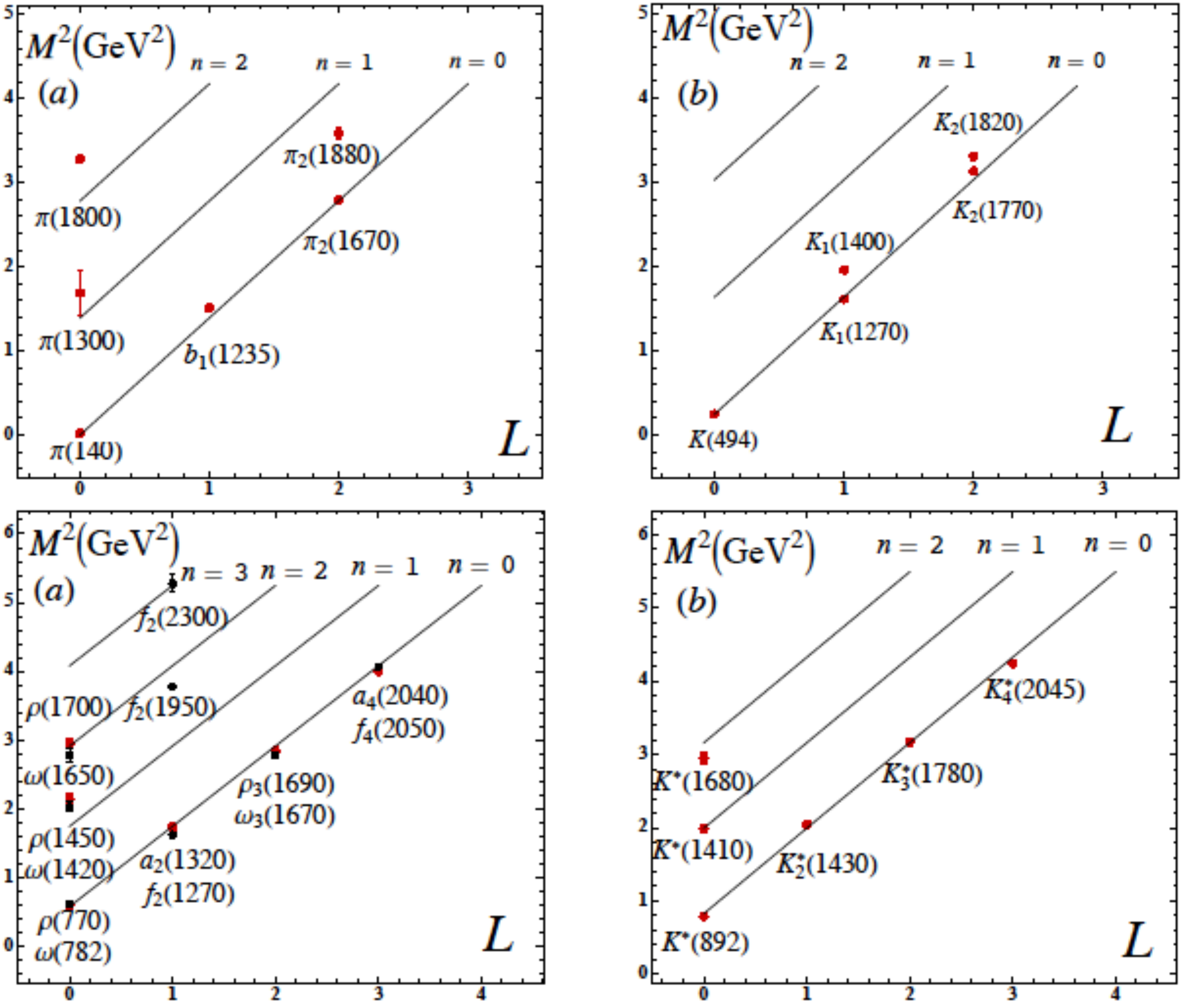
$$m_q = 46\ MeV,\ m_s = 357\ MeV$$

Orbital and Radial Excitations

De Teramond, Dosch, sjb

$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$

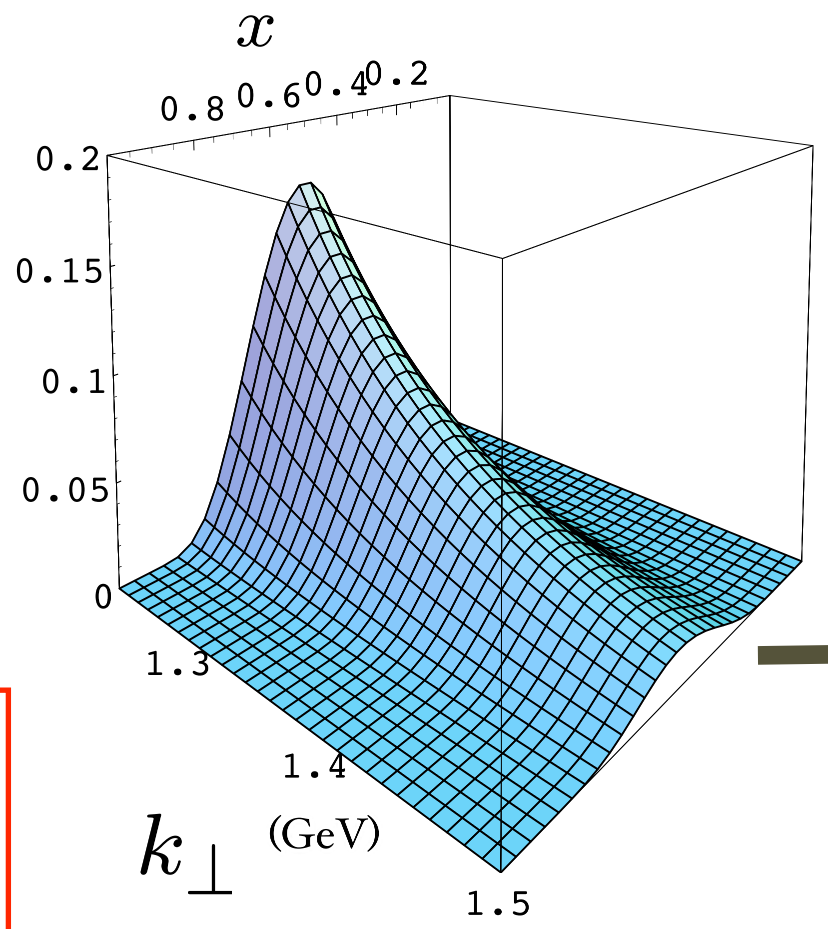
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$



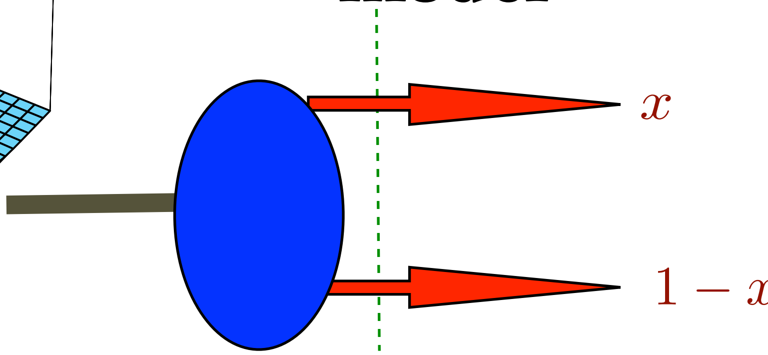
Prediction from AdS/QCD: Meson LFWF

de Teramond, Cao, sjb

$\psi_M(x, k_\perp^2)$



“Soft Wall” model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

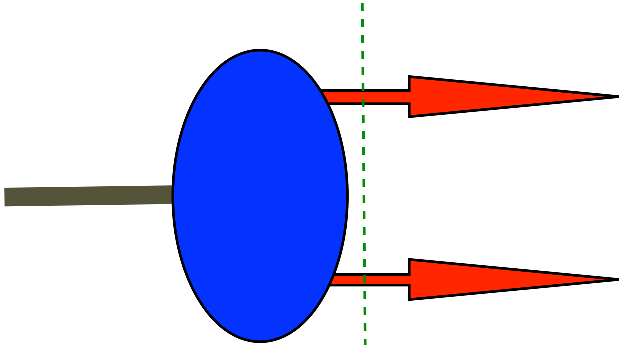
$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Remarkable Features of Light-Front Schrödinger Equation

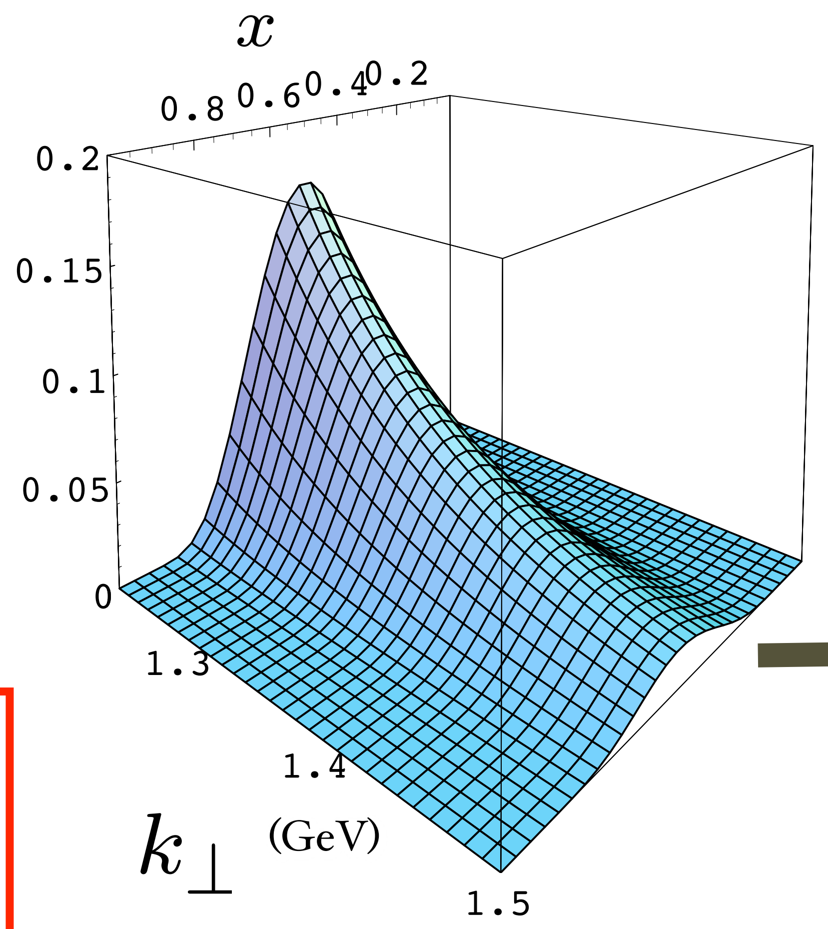
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Prediction from AdS/QCD: Meson LFWF

de Teramond, Cao, sjb

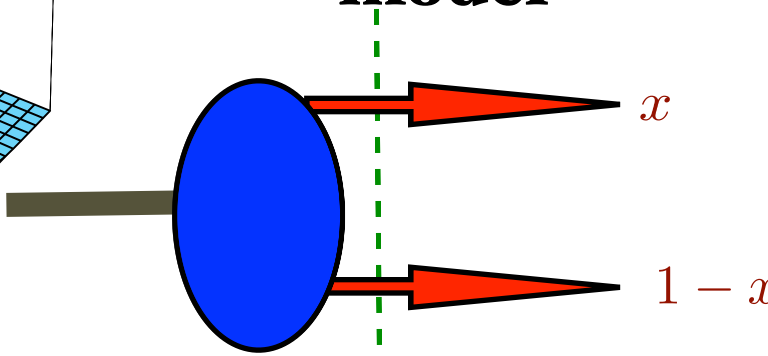
$\psi_M(x, k_\perp^2)$



Note coupling

$k_\perp^2, x$

"Soft Wall" model



massless quarks

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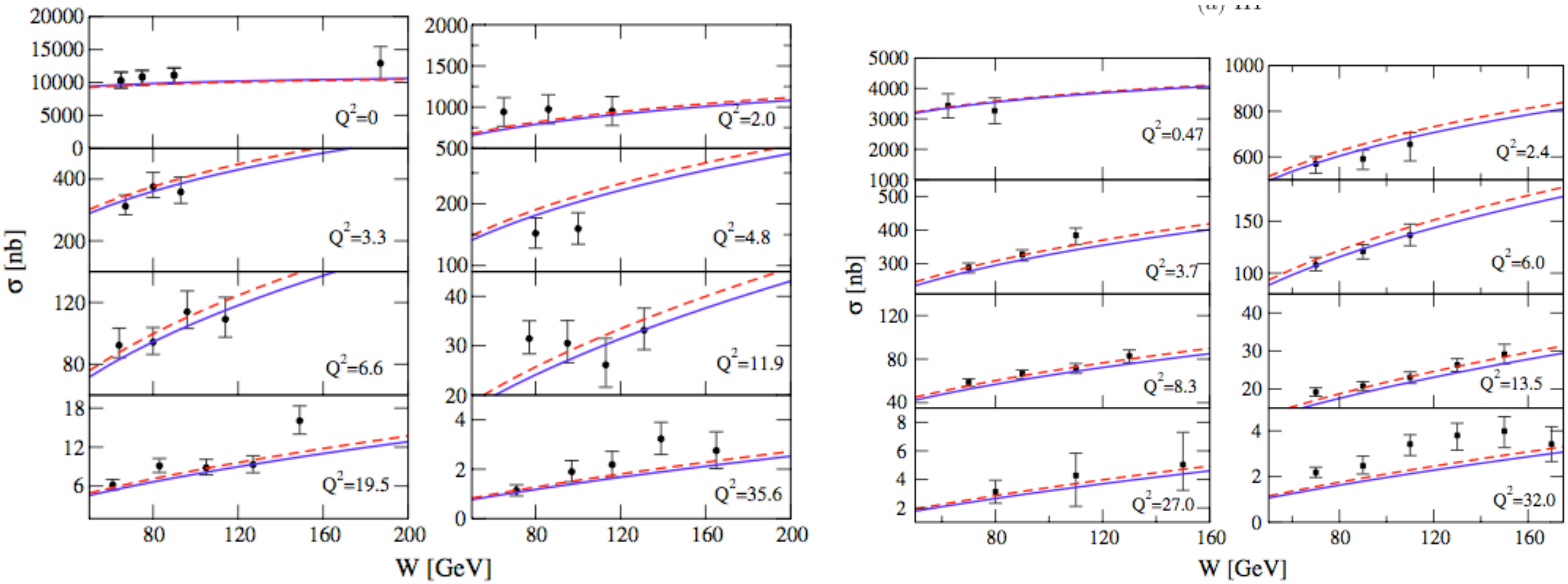
Provides Connection of Confinement to Hadron Structure

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



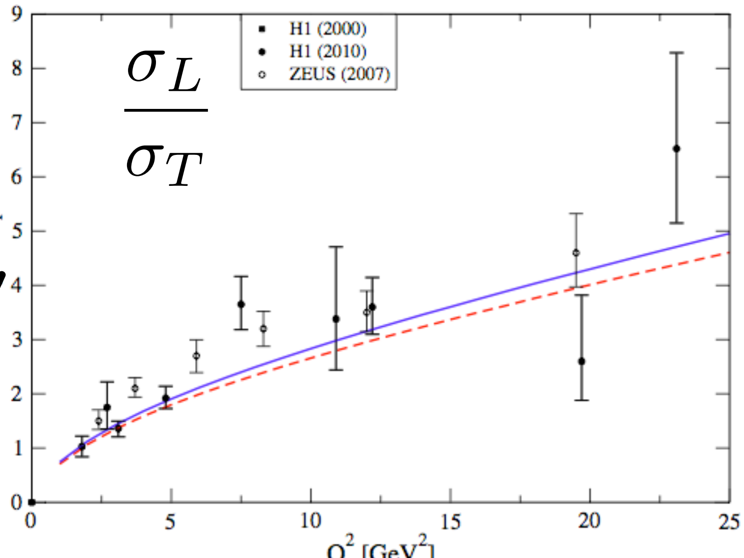
(a) H1

(b) ZEUS

J. R. Forshaw,
R. Sandapen

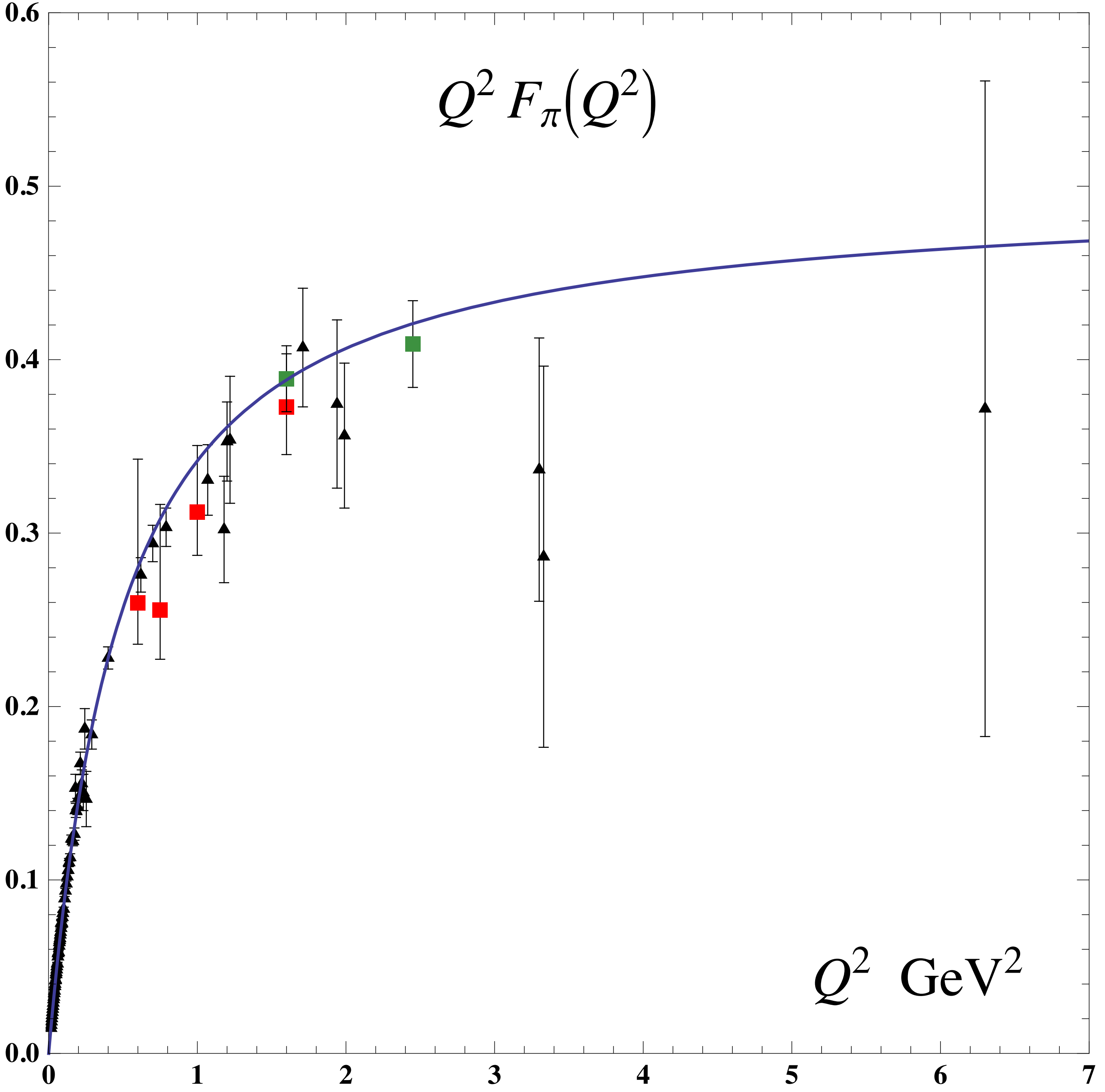
$$\gamma^* p \rightarrow \rho^0 p'$$

$$\frac{\sigma_L}{\sigma_T}$$

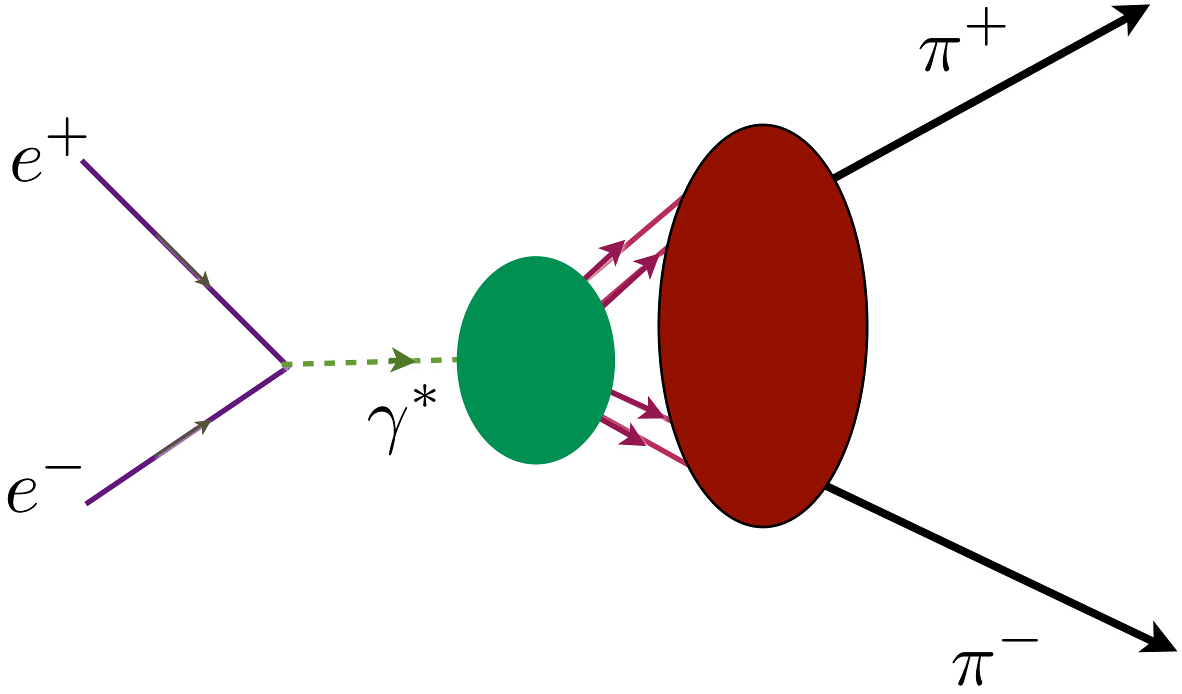


Prediction from
Light-Front Holography

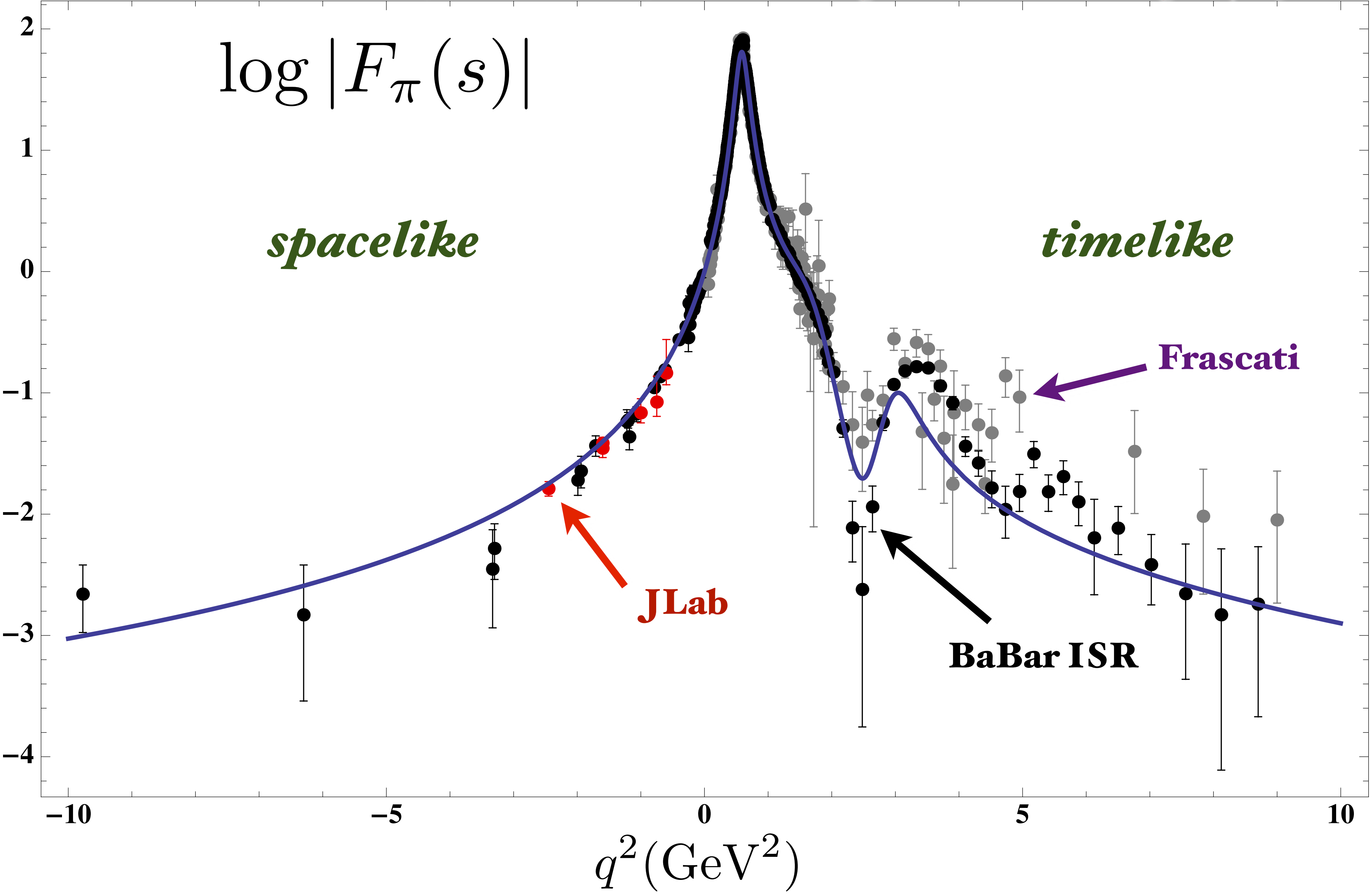
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

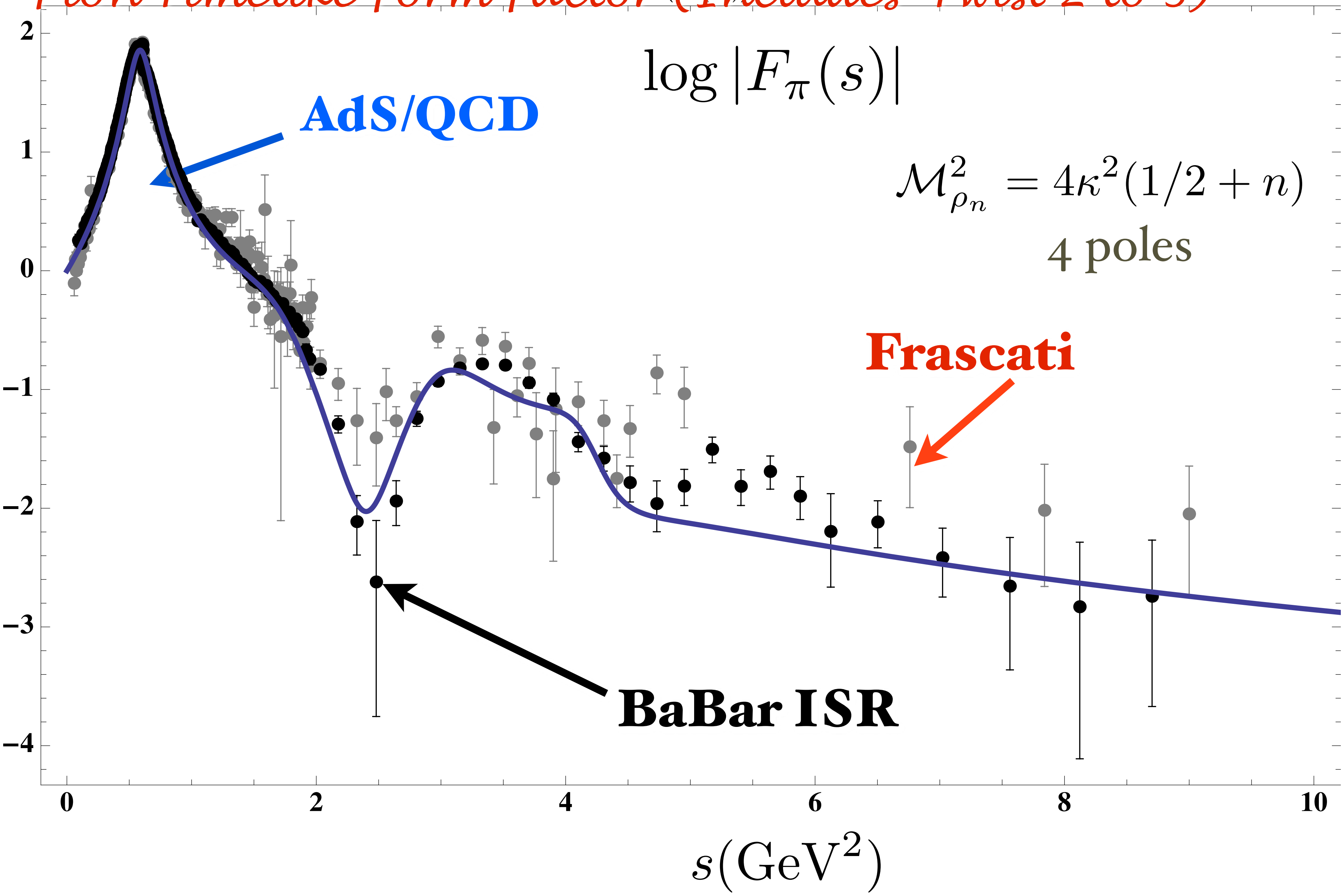
Dressed soft-wall current brings in higher Fock states and more vector meson poles



Pion Form Factor from AdS/QCD and Light-Front Holography



Pion Timelike Form Factor (Includes Twist 2 to 5)

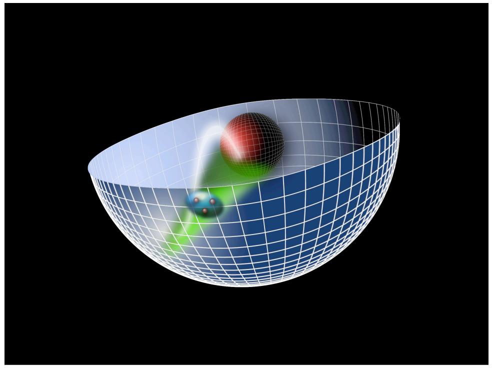


de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

Single scheme-independent fundamental mass scale

κ



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry
of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

(m$_q$=0)

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Bjorken sum rule defines effective charge

$\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Bjorken sum rule defines effective charge

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

$\alpha_{g1}(Q^2)$

- Can be used as standard QCD coupling
- Well measured
- Asymptotic freedom at large Q^2
- Computable at large Q^2 in any pQCD scheme
- Universal β_0, β_1

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

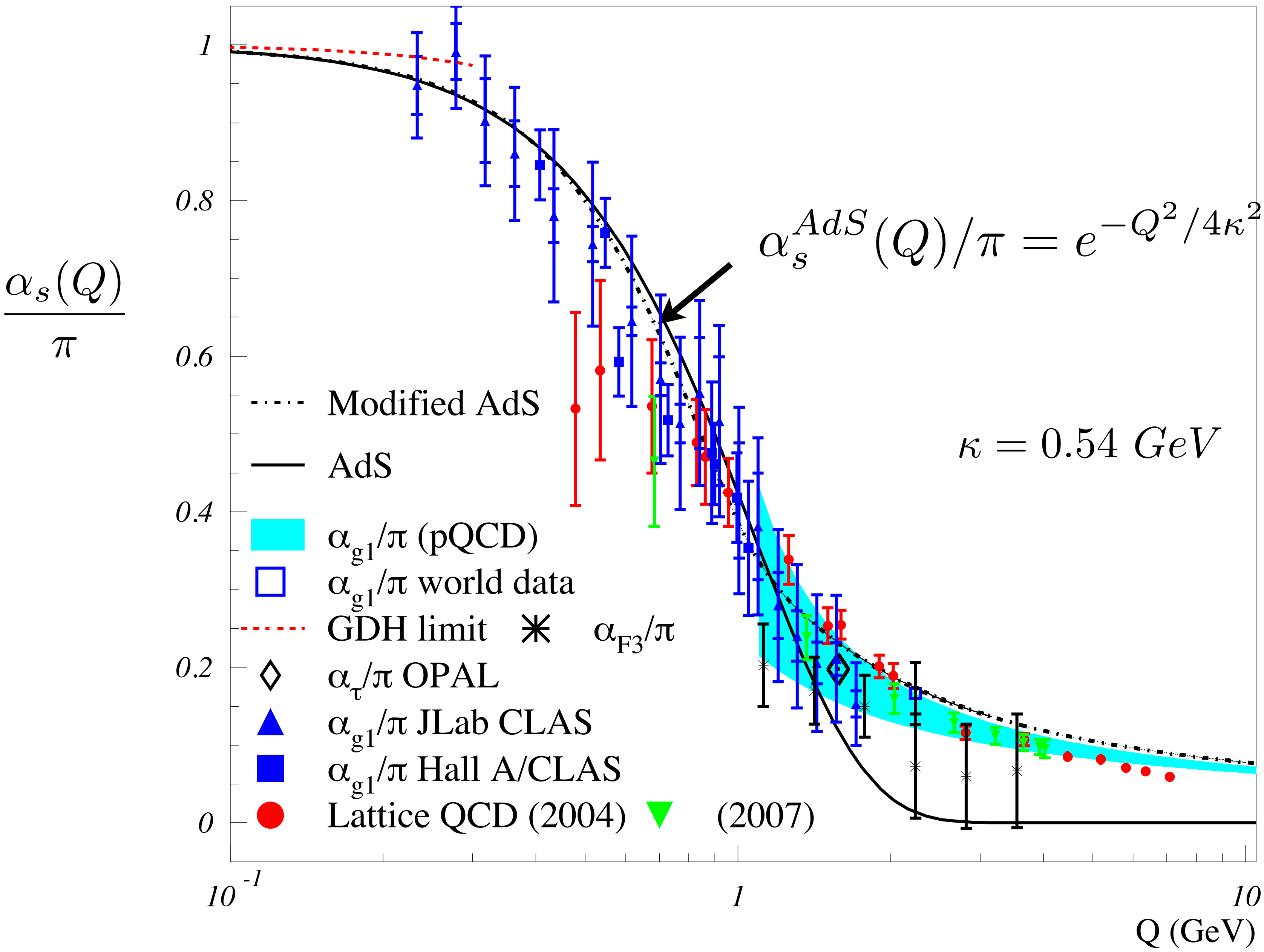
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

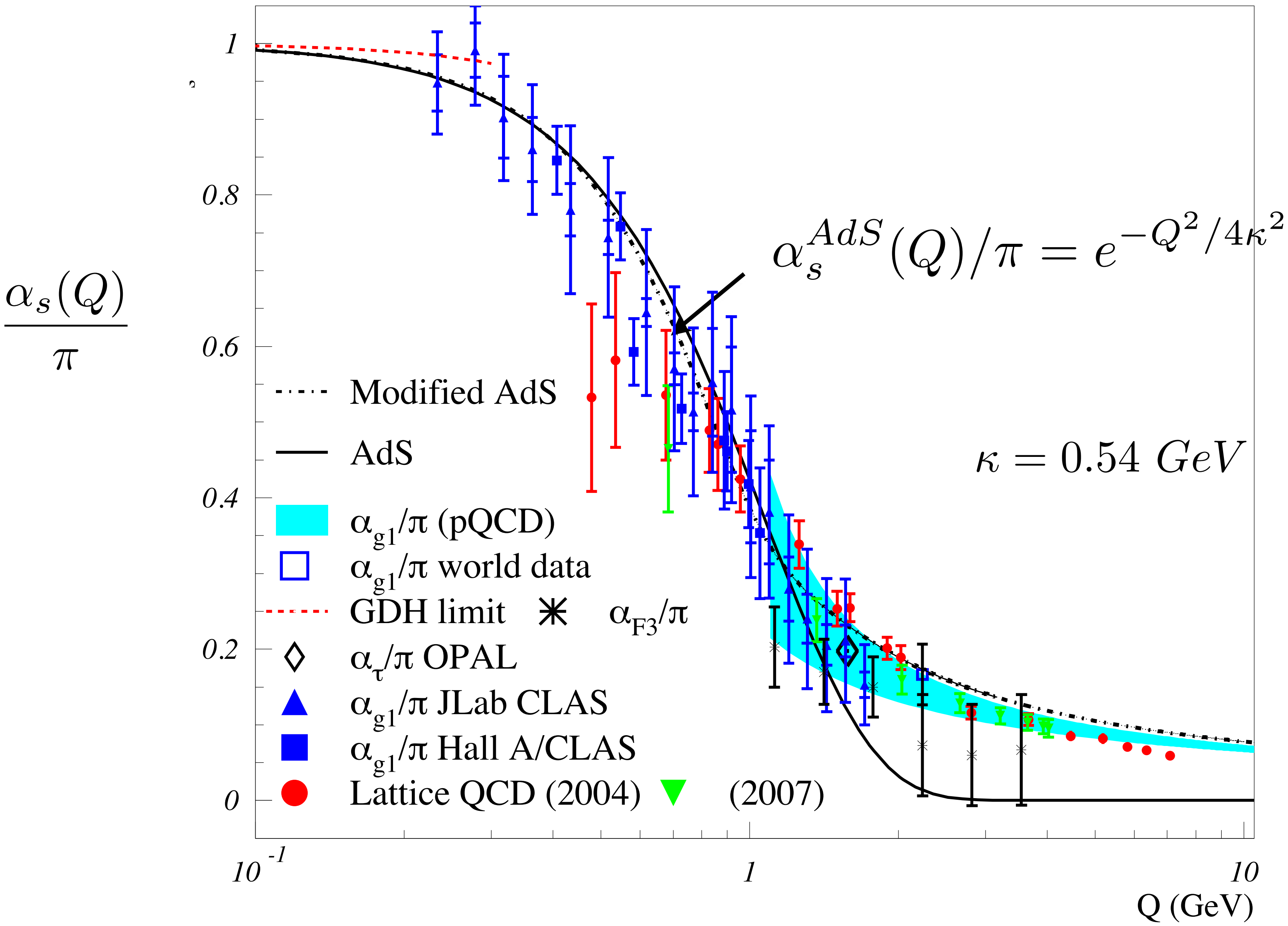


$$\alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2}$$

$\kappa = 0.54\ GeV$

Deur, de Teramond, sjb

Analytic, defined at all scales, IR Fixed Point



$$\alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2}$$

$$\kappa = 0.54\ GeV$$

AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

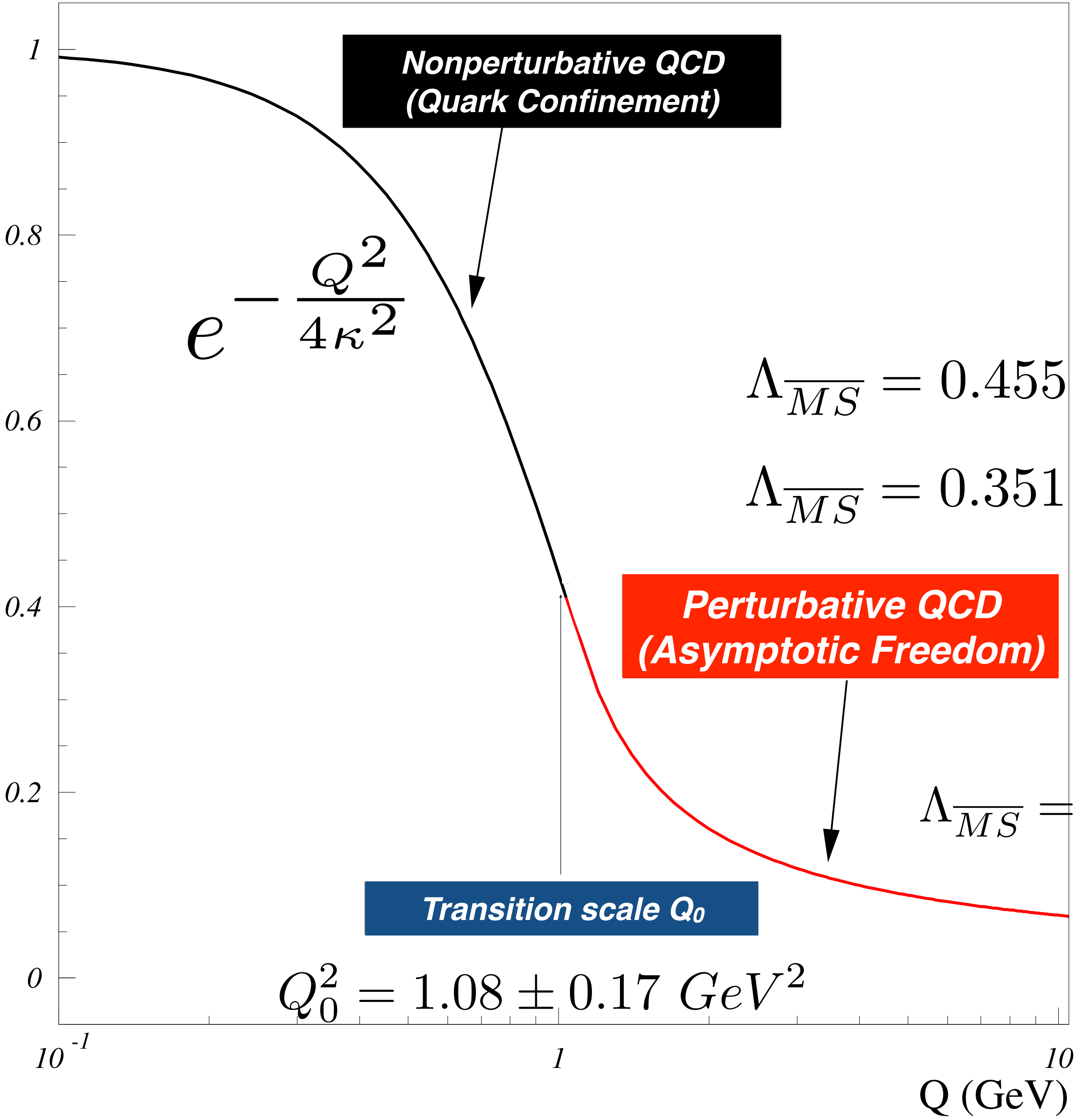
$$m_\rho = \sqrt{2}\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

Prediction from AdS/QCD:

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$$e^{-\frac{Q^2}{4\kappa^2}}$$

Nonperturbative QCD (Quark Confinement)

Perturbative QCD (Asymptotic Freedom)

Transition scale Q₀

$$Q_0^2 = 1.08 \pm 0.17\ GeV^2$$

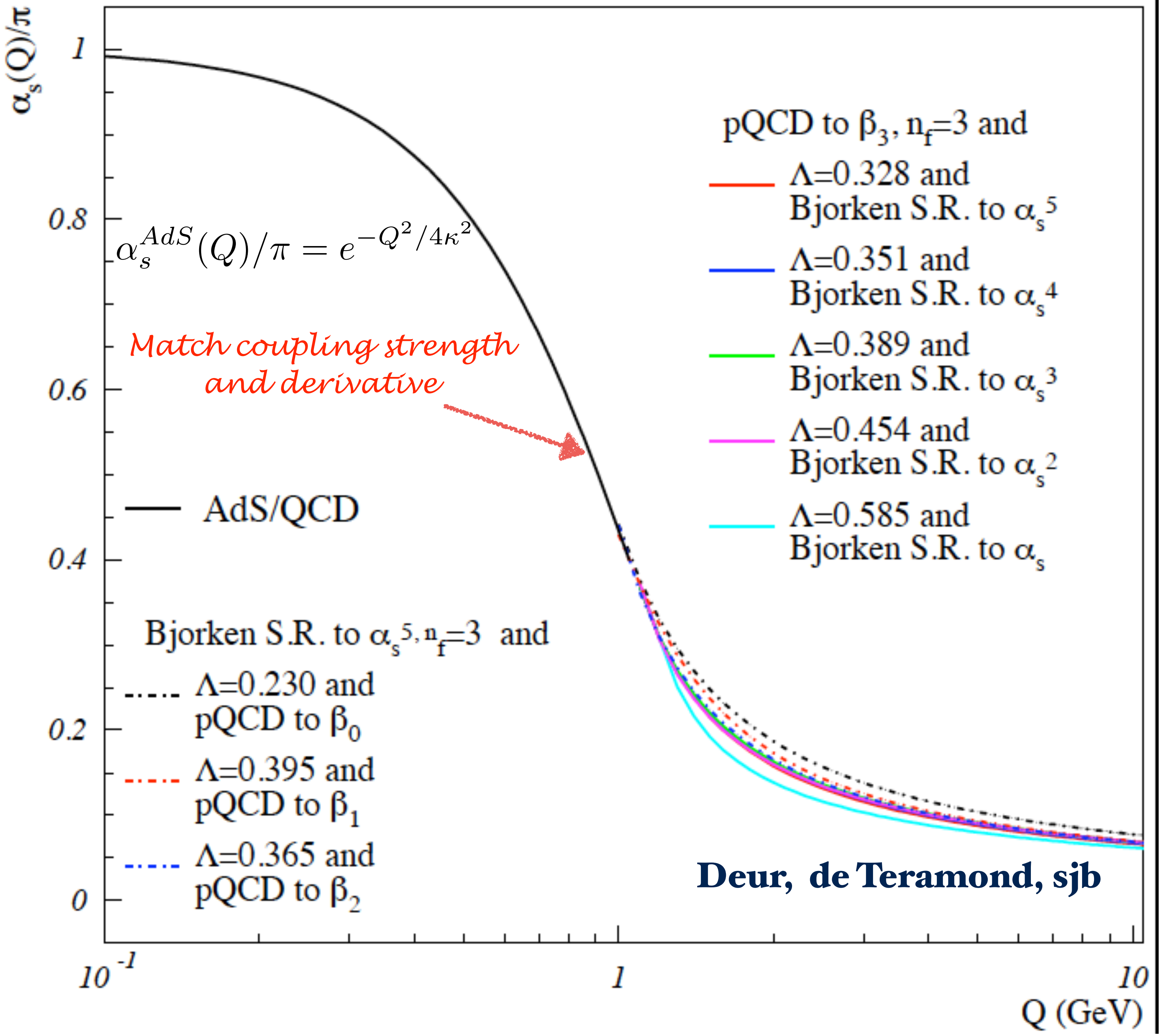
$$\Lambda_{\overline{MS}} = 0.455 \pm 0.031 M_\rho$$

$$\Lambda_{\overline{MS}} = 0.351 \pm 0.024\ GeV$$

Expt:

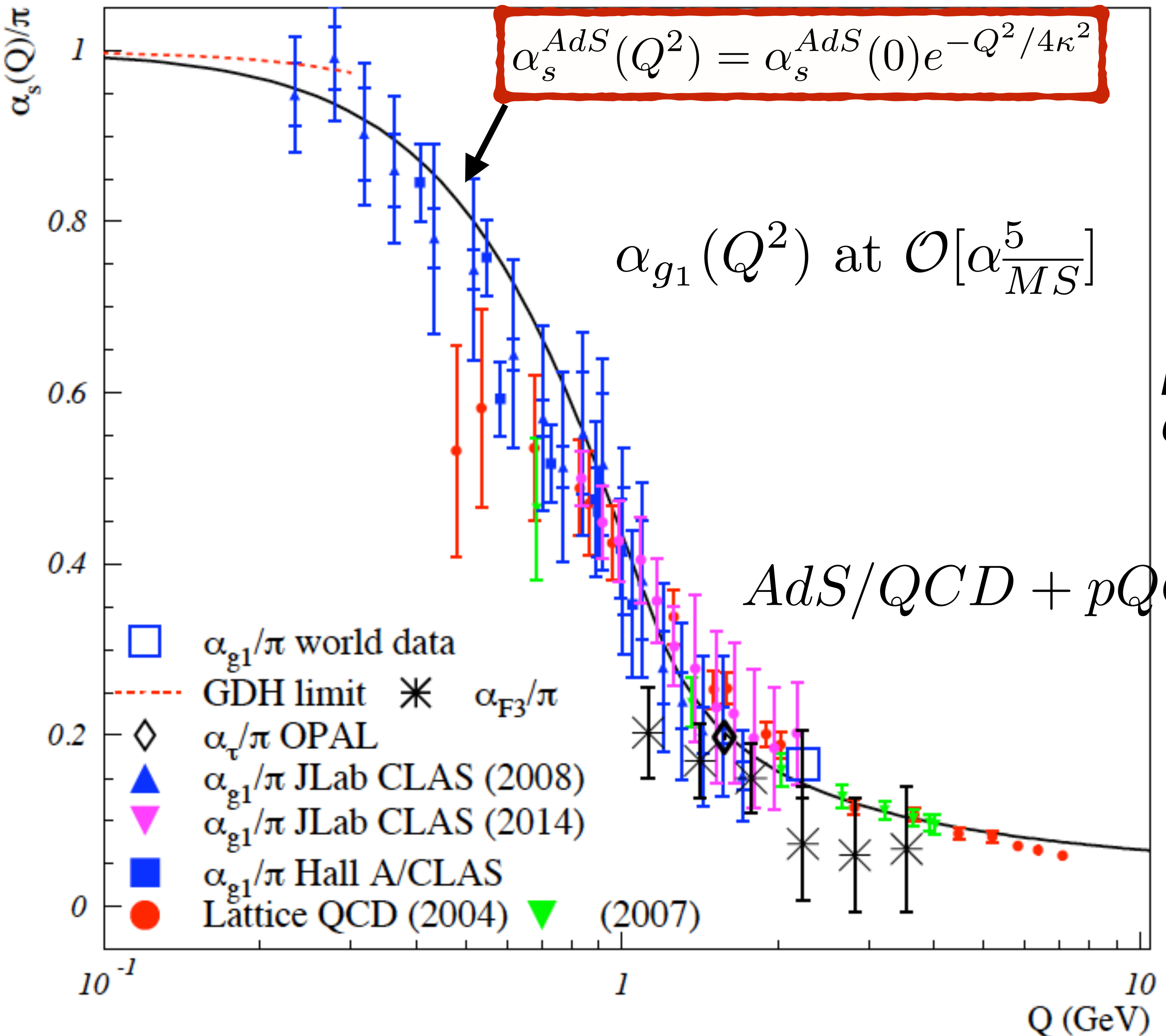
$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024\ G$$

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur, de Teramond, sjb

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}$$

$\alpha_{g_1}(Q^2)$ at $\mathcal{O}[\alpha^5_{\overline{MS}}]$

$AdS/QCD + pQCD_{\mathcal{O}(\beta_3)}$

Deur,
de Teramond, sjb

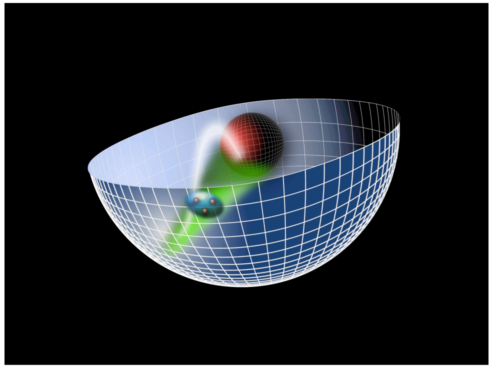
Matching analysis:
Courtoy and Liuti

Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry
of the action

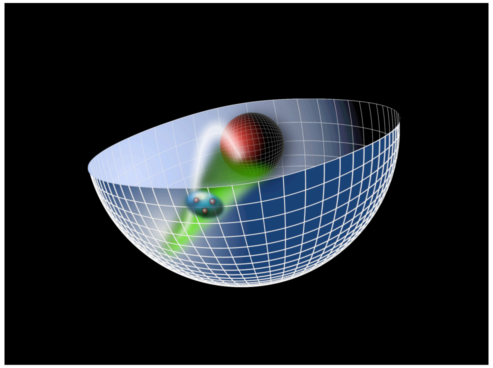
Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:** **Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!**

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal Symmetry
of the action

Light-Front Schrödinger Equation

Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion (m_q =0)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S.**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit required**

Uniqueness

de Teramond, Dosch, sjb

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- ζ_2 **confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial \tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\Big(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\Big)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)

- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$

- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $4uw - v^2 = \kappa^4 = [M]^4$

- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ

- Only dimensionless mass ratios (and M times R) predicted

- Mass and time units [GeV] and [sec] from physics external to QCD

- New feature: bounded frame-independent relative time

dAFF: New Time Variable

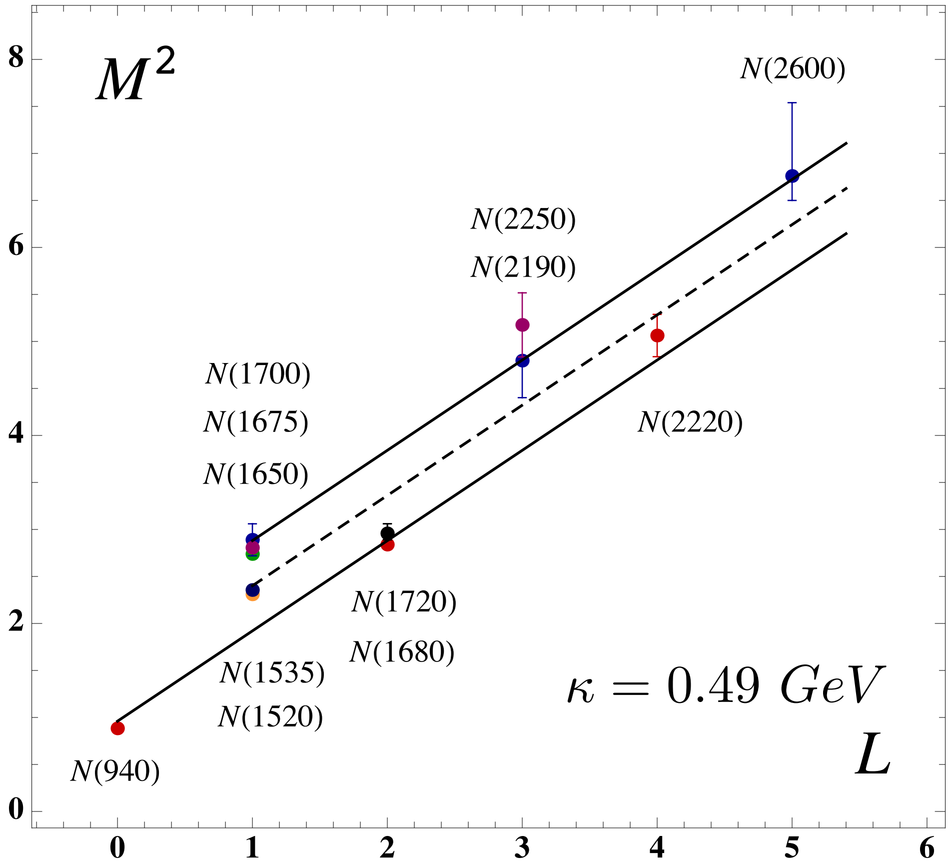
$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

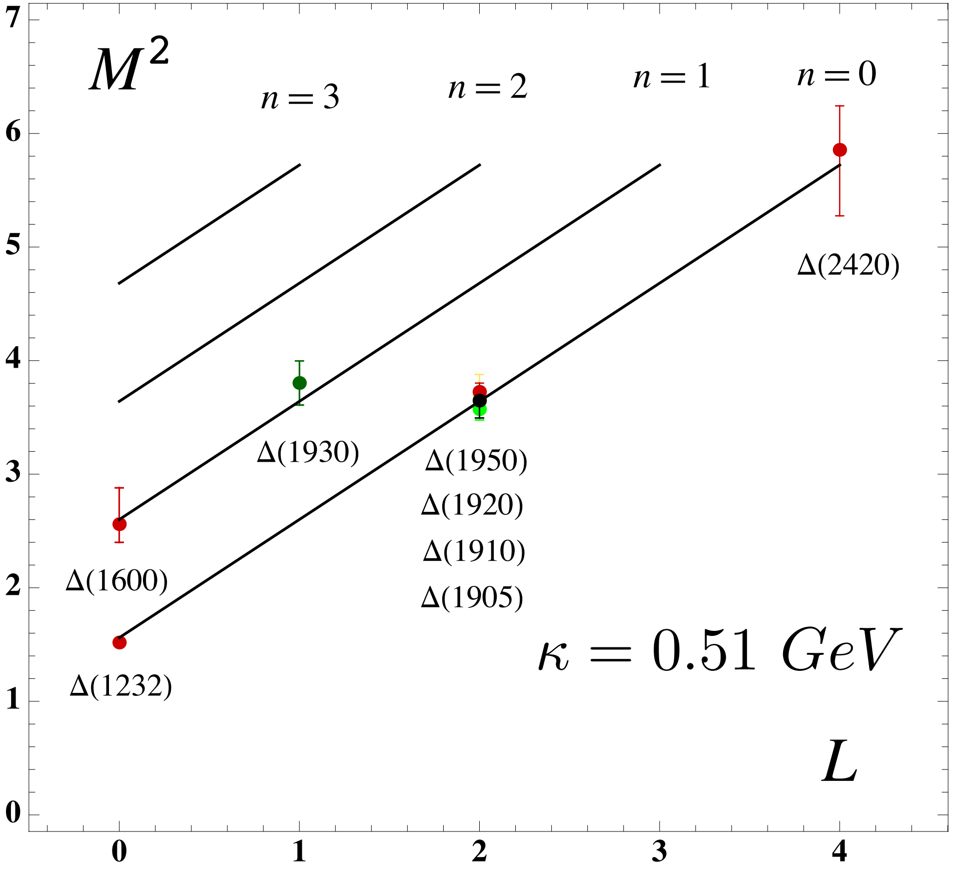
- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double Parton Processes**

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Baryon Spectroscopy from AdS/QCD and Light-Front Holography





de Teramond, sjb

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$

positive parity

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$

negative parity

All confirmed resonances from PDG 2012

See also Forkel, Beyer, Federico, Klempt

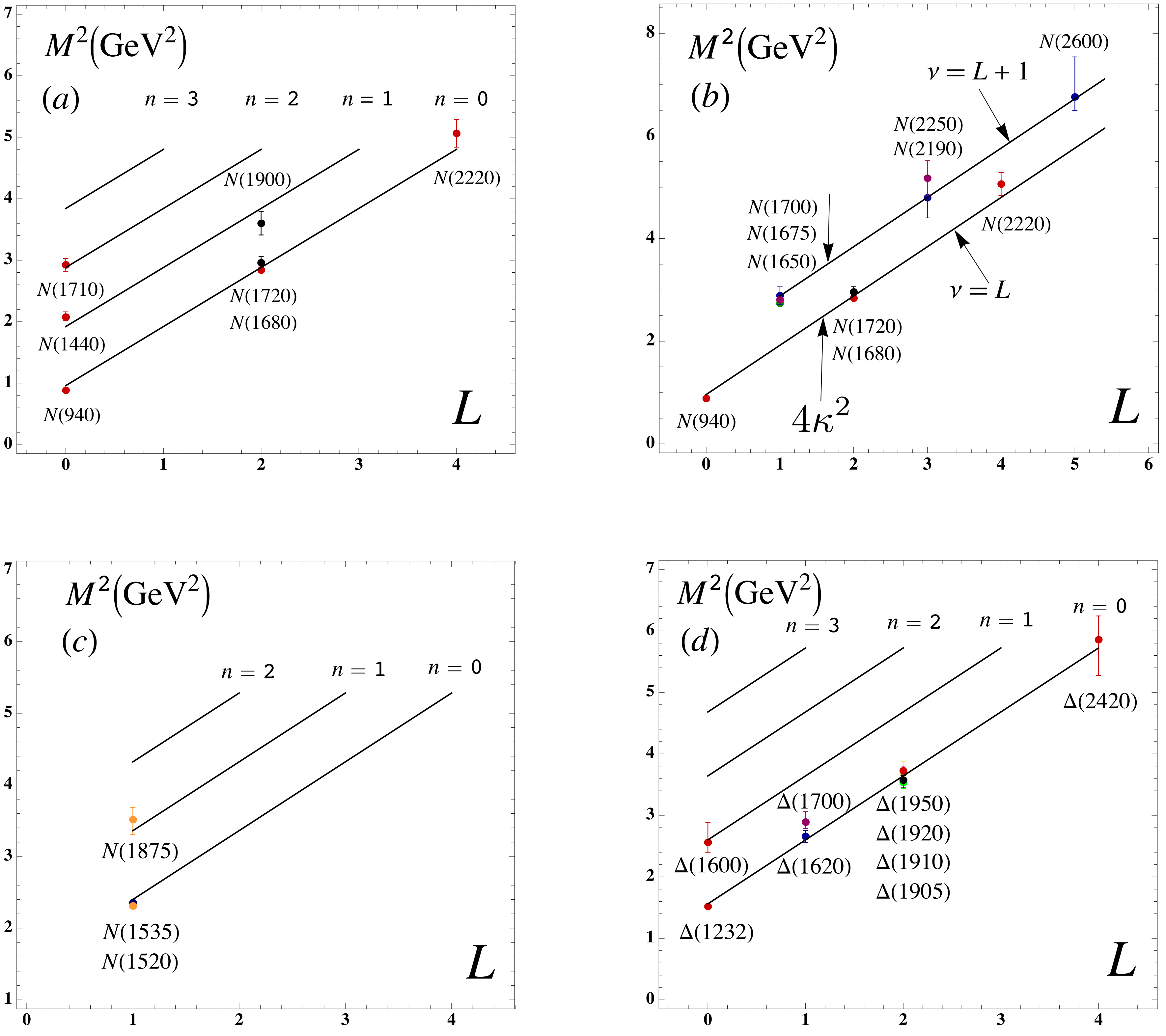

$M^2(\mathrm{GeV}^2)$
(a)
$n=3$
$n=2$
$n=1$
$n=0$
N(2220)
N(1900)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
0 1 2 3 4 5 6 7
0 1 2 3 4
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L+1$
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
L
0 2 4 6 8
0 1 2 3 4 5 6
$M^2(\mathrm{GeV}^2)$
(c)
$n=2$
$n=1$
$n=0$
N(1875)
N(1535)
N(1520)
L
1 2 3 4 5 6 7
0 1 2 3 4
$M^2(\mathrm{GeV}^2)$
(d)
$n=3$
$n=2$
$n=1$
$n=0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
L
0 1 2 3 4 5 6 7
0 1 2 3 4

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

PDG 2012

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

LF Holography

Baryon Equation

$$x \to \zeta$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ \quad = \quad M^2\,\psi_J^+,$$

G22

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- \quad = \quad M^2\,\psi_J^-.$$

G11

$$M_B^2(n, L_B) = 4\lambda_B^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M\,(J-1) + \frac{4\nu^2-1}{4\zeta^2}\right)\phi_J = M^2\,\phi_J,$$

G11

$$M_M^2(n, L_M, S=0) = 4\lambda_M^2(n + L_M) \qquad \nu = L_M$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

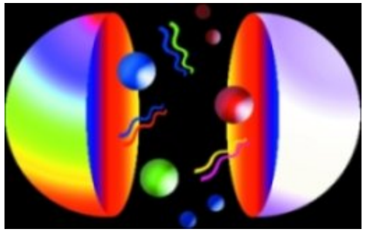
Meson-Baryon Degeneracy for $L_M=L_B+1$

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\,(n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Superconformal Algebra

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[\mathrm{H,D}] = \mathrm{i\ H}, \quad [\mathrm{H,\ K}] = 2\ \mathrm{i\ D}, \quad [\mathrm{K,\ D}] = -\ \mathrm{i\ K}$$

Fubini and Rabinovici

Superconformal Algebra

de Teramond Dosch and SJB

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\boxed{Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS;$ $\quad$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \big)$$

$$G_{22} = \big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \big)$$

Identify $f - \frac{1}{2} = L_B \;, \quad w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

$$x \to \zeta$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ \;=\; M^2\,\psi_J^+,$$

G22

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- \;=\; M^2\,\psi_J^-.$$

G11

$$M_B^2(n, L_B) = 4\lambda_B^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M\,(J-1) + \frac{4\nu^2-1}{4\zeta^2}\right)\phi_J = M^2\,\phi_J,$$

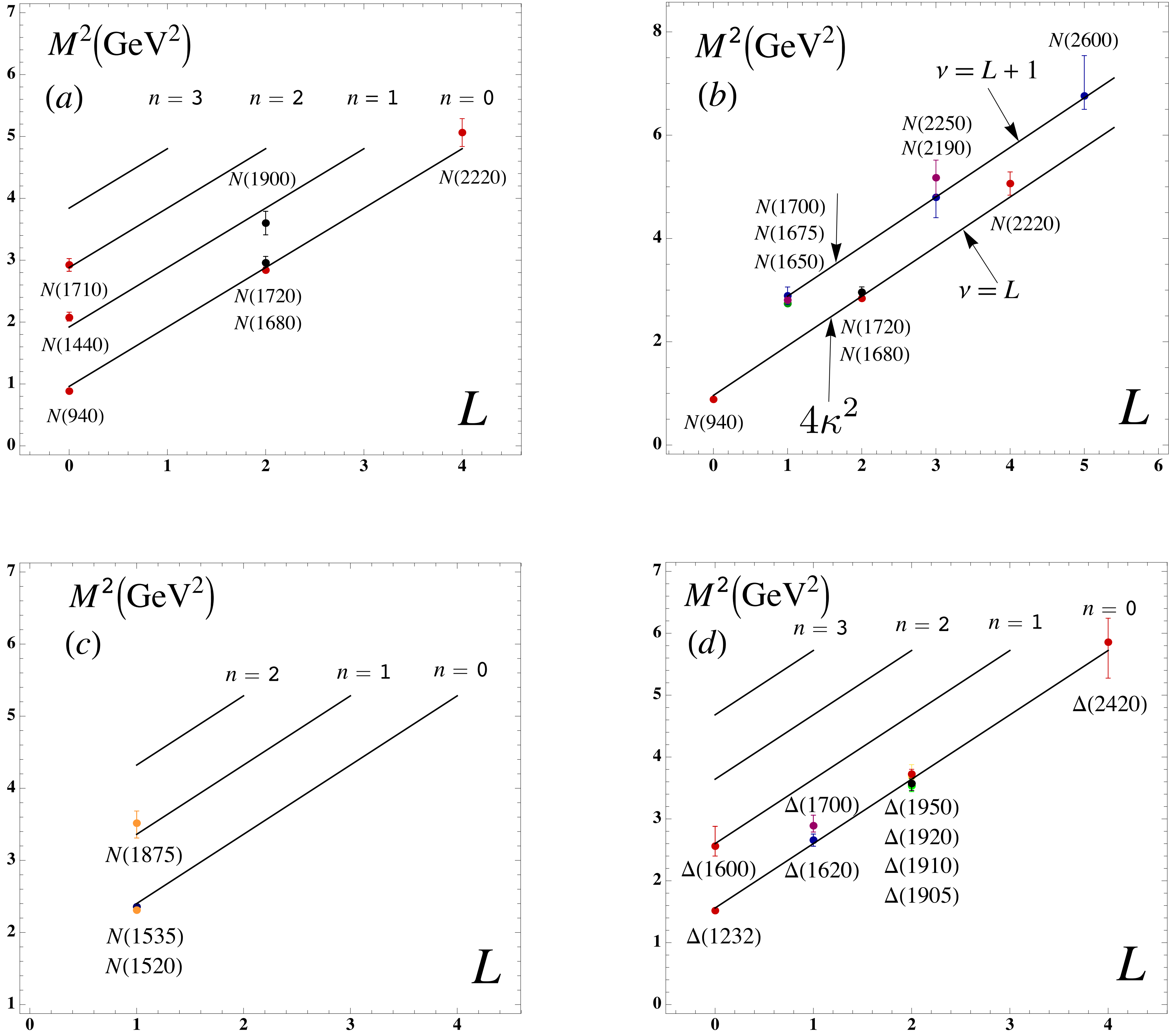
G11

$$M_M^2(n, L_M, S=0) = 4\lambda_M^2(n + L_M) \qquad \nu = L_M$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$


$M^2(\mathrm{GeV}^2)$
(a)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
0 1 2 3 4 5 6 7
0 1 2 3 4
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
N(1700)
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N(1680)
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Δ(1620)
Δ(1232)
L
0 1 2 3 4 5 6 7
0 1 2 3 4

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$\boxed{J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0}$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad ({F_1}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

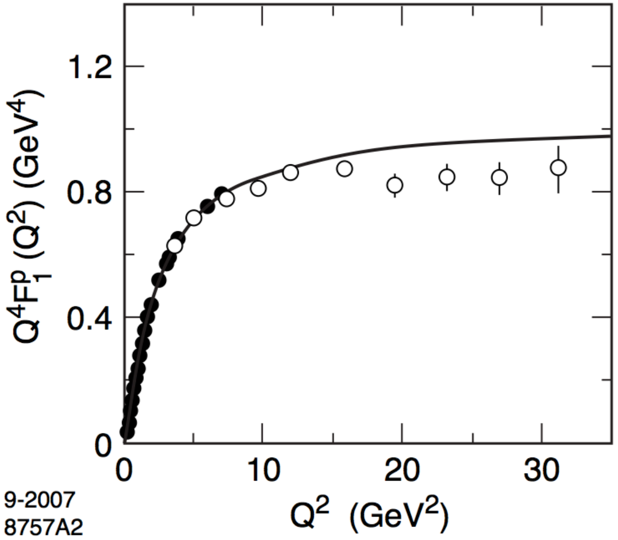
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} \, e^{-\kappa^2 z^2 x/(1-x)}$$

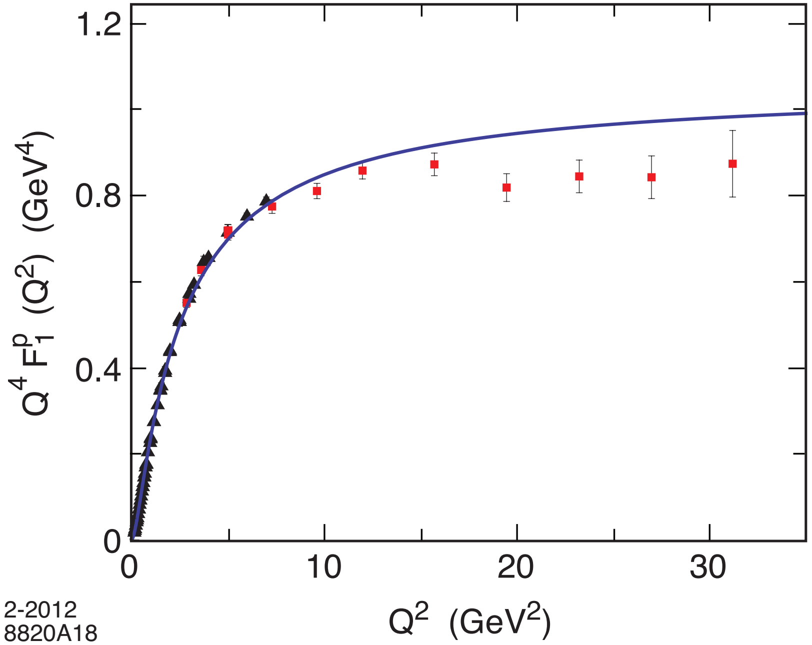
- Find

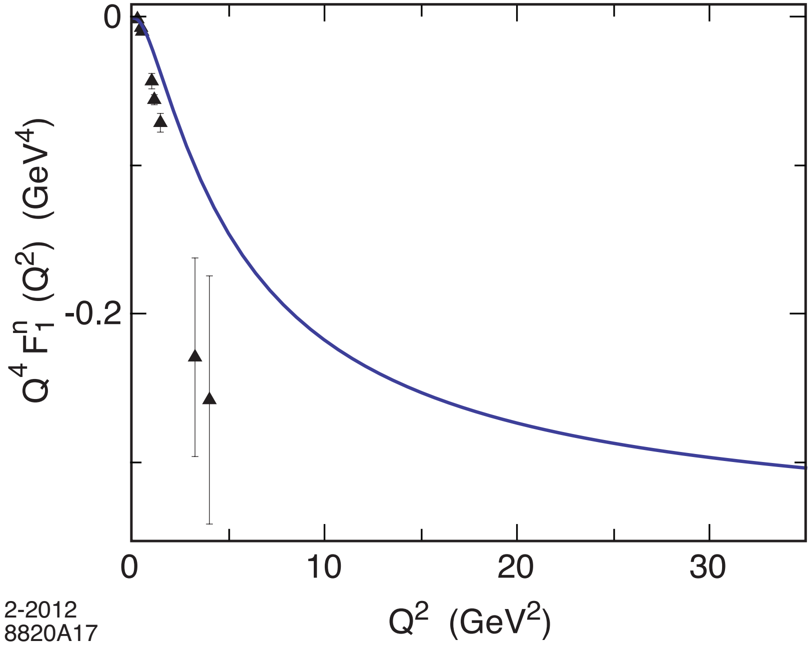
$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

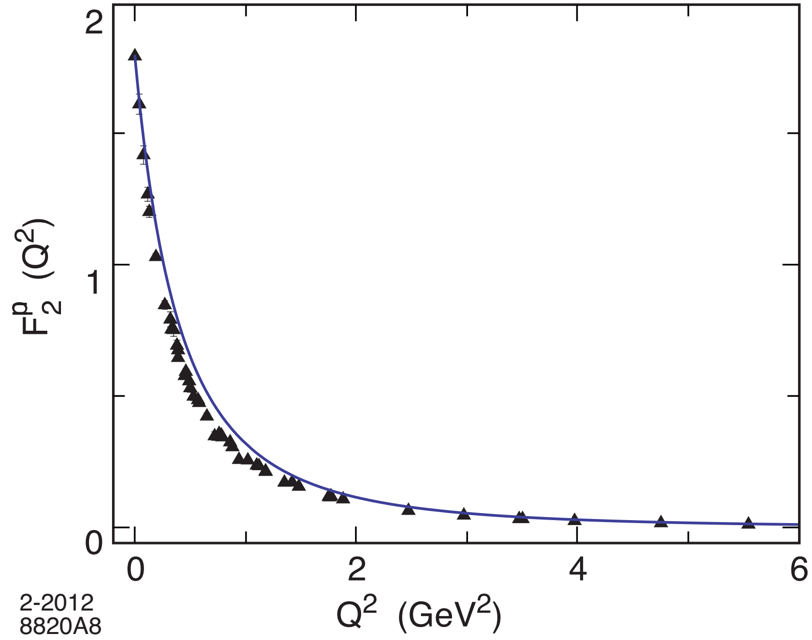
with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

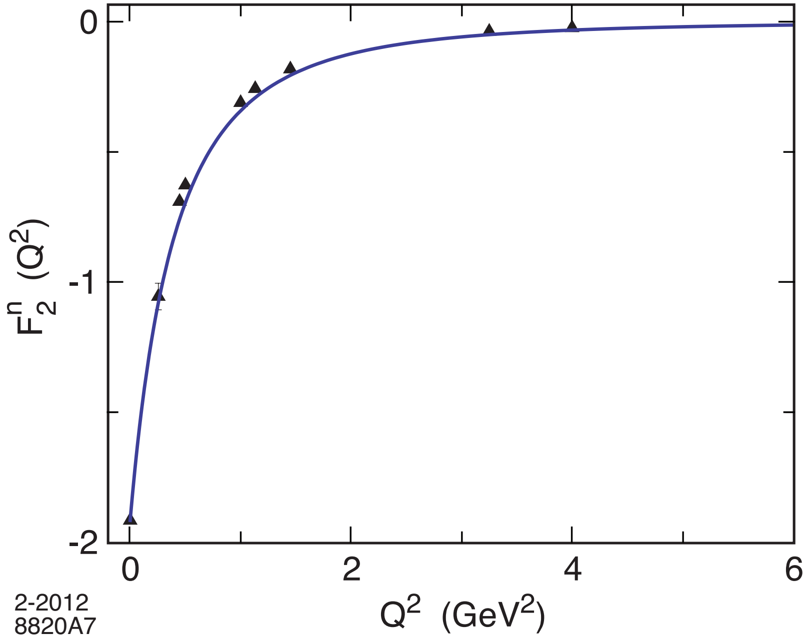


Using $SU(6)$ flavor symmetry and normalization to static quantities



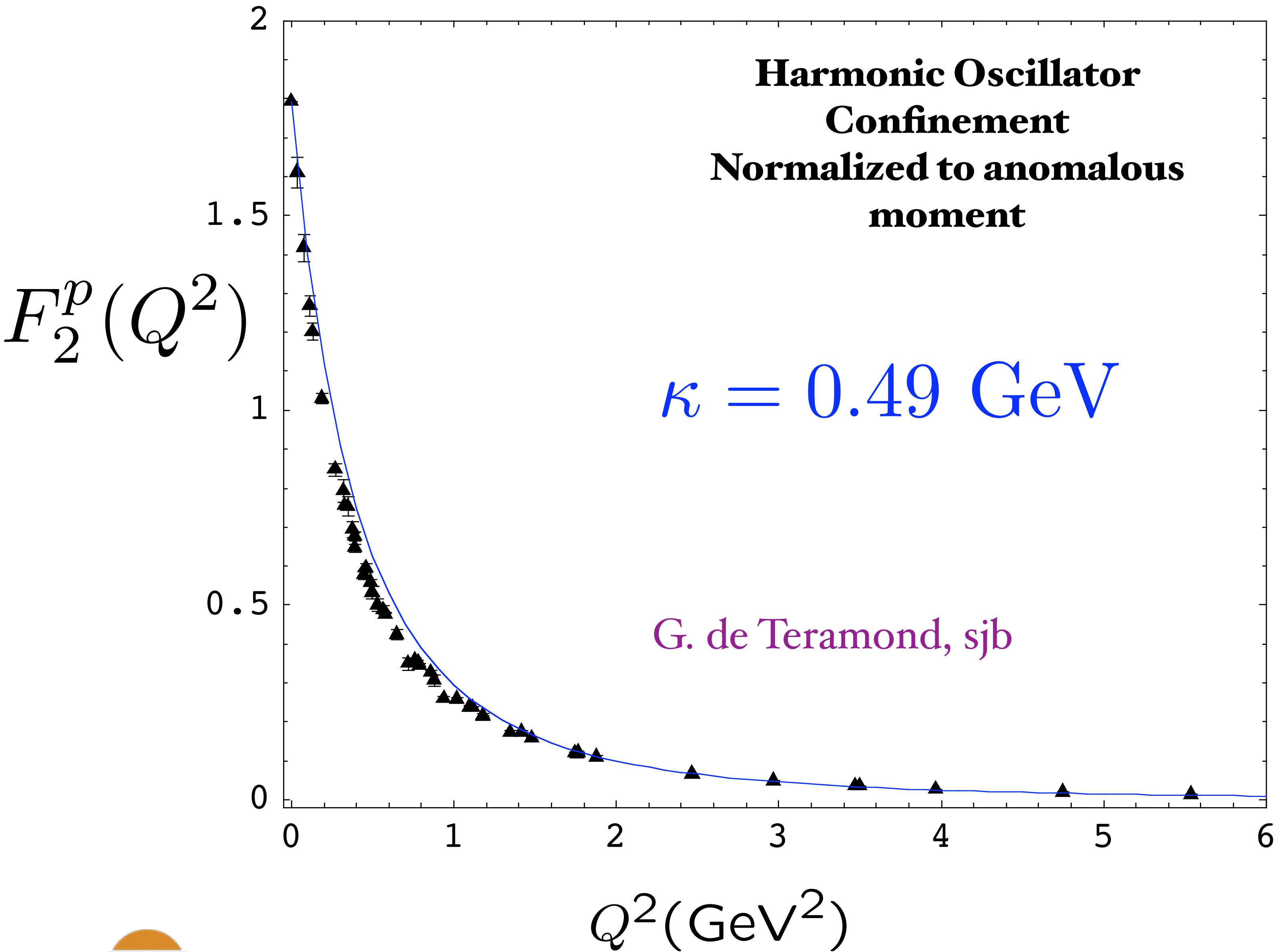






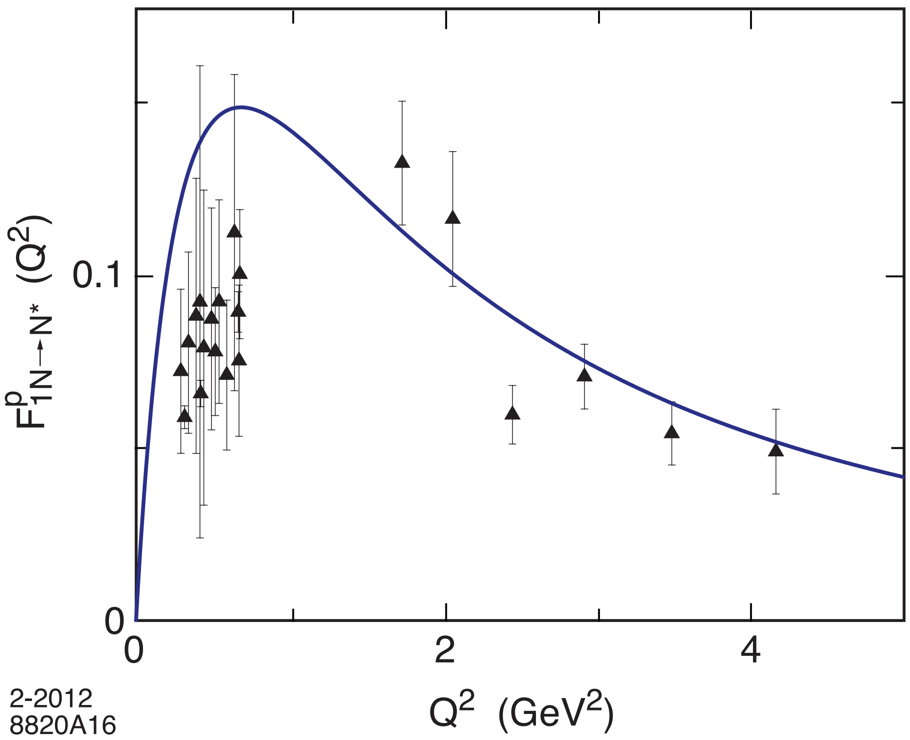
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

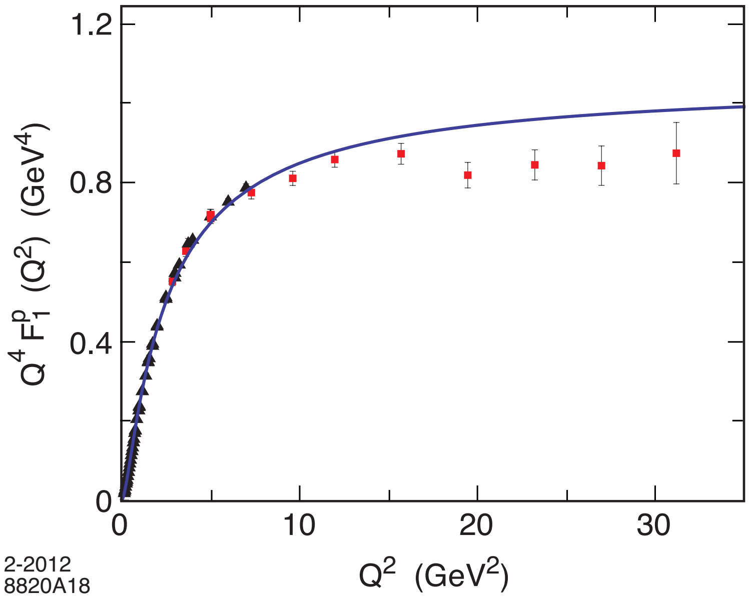
- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

Predictions for nucleon form factors from AdS/QCD

Using $SU(6)$ flavor symmetry and normalization to static quantities



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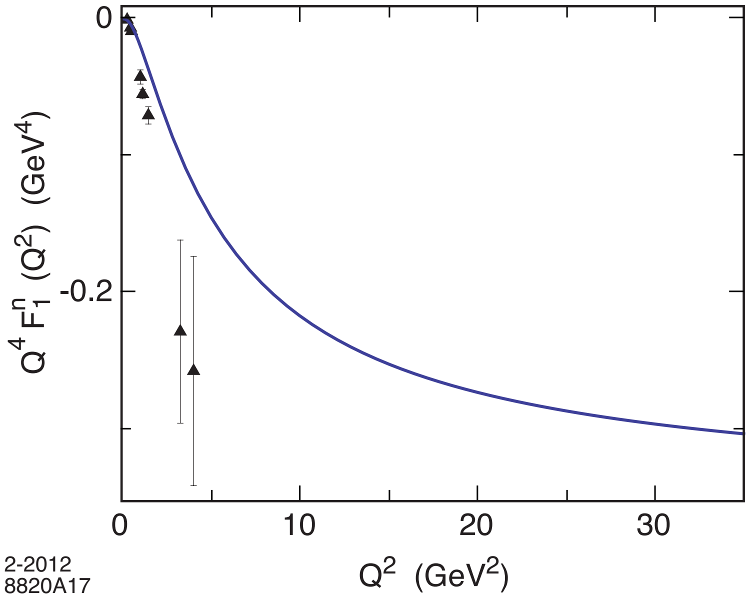
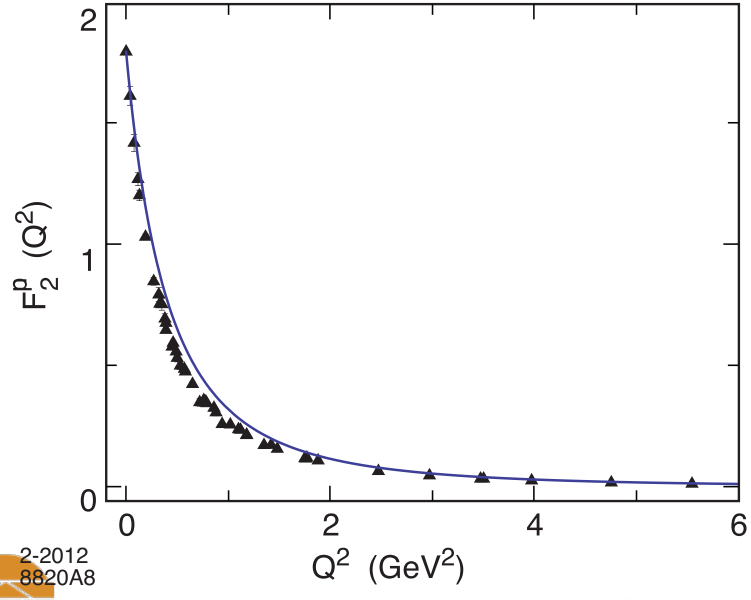
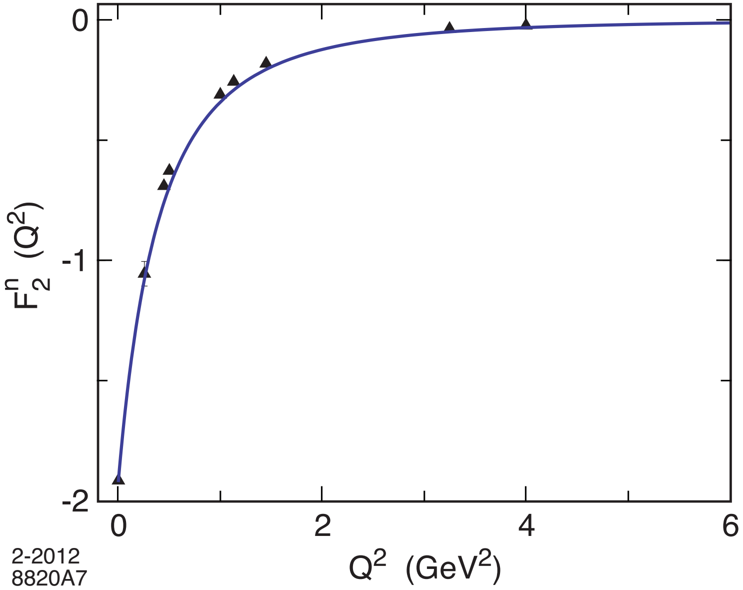


2-2012
8820A17



2-2012
8820A8

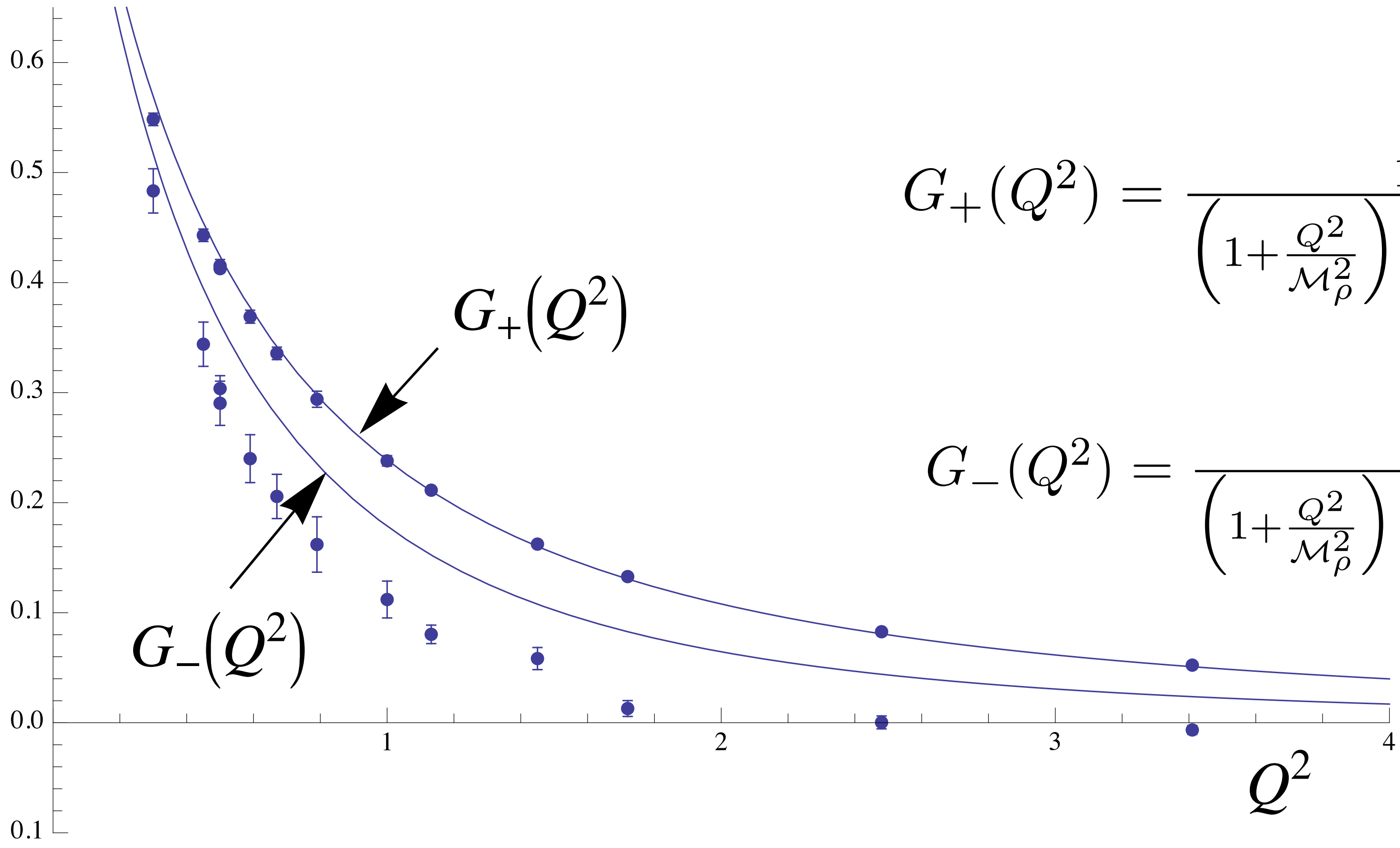


2-2012
8820A7

Flavor Decomposition of Elastic Nucleon Form Factors

G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-, \qquad F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-, \qquad F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$



$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{\;p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\;p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\;p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

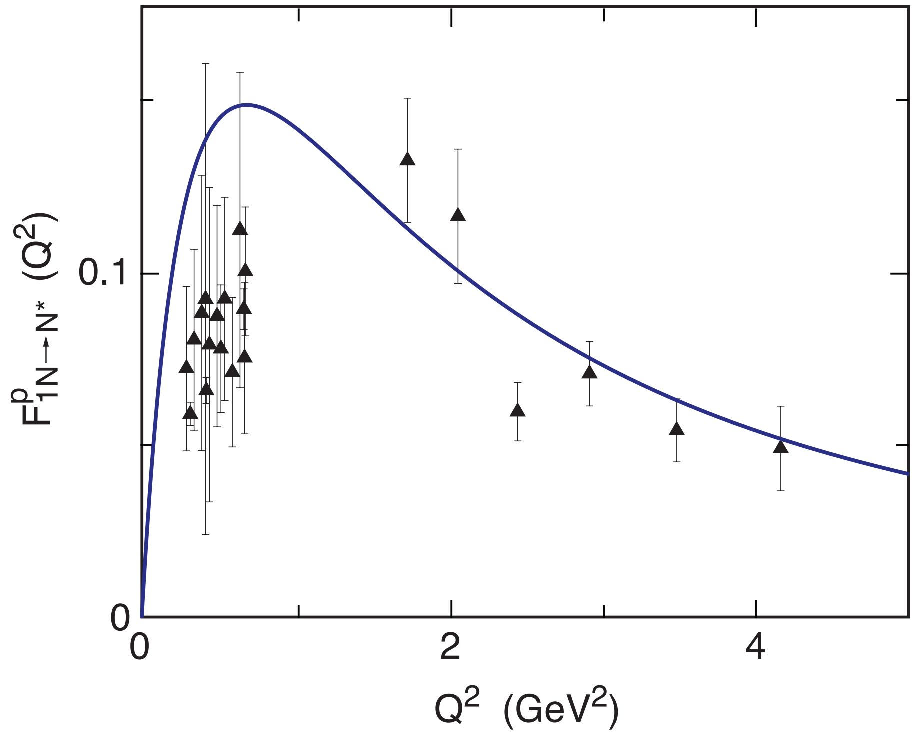
de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$

AdS\QCD
Light-Front
Holography



G. de Teramond, sjb

Proton transition form factor to the first radial excited state. Data from JLab

LF Holography

Baryon Equation

$$x \to \zeta$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ \quad = \quad M^2\,\psi_J^+,$$

G22

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- \quad = \quad M^2\,\psi_J^-.$$

G11

$$M_B^2(n, L_B) = 4\lambda_B^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M\,(J-1) + \frac{4\nu^2-1}{4\zeta^2}\right)\phi_J = M^2\,\phi_J,$$

G11

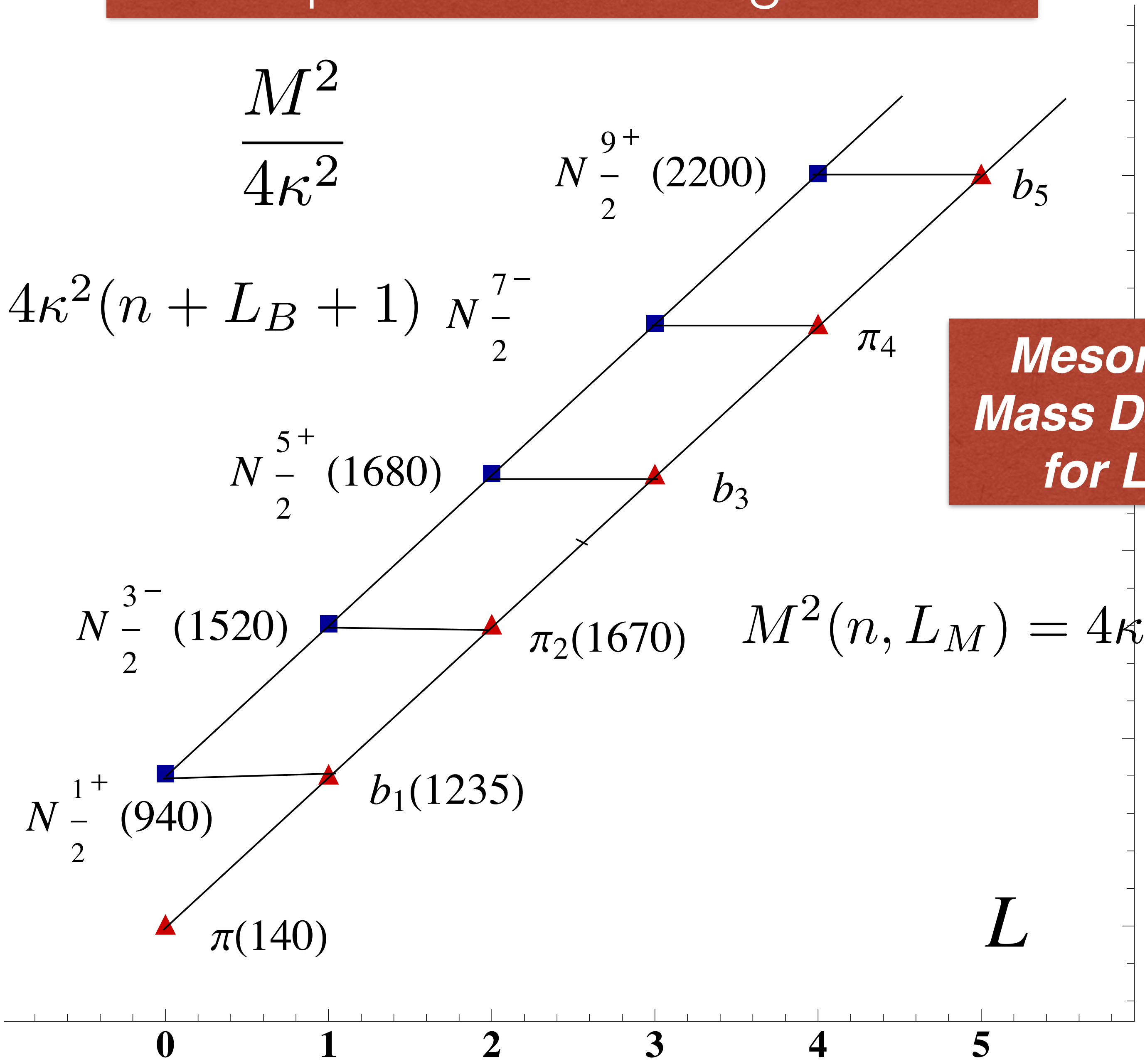
$$M_M^2(n, L_M, S=0) = 4\lambda_M^2(n + L_M) \qquad \nu = L_M$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

Superconformal Algebra



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

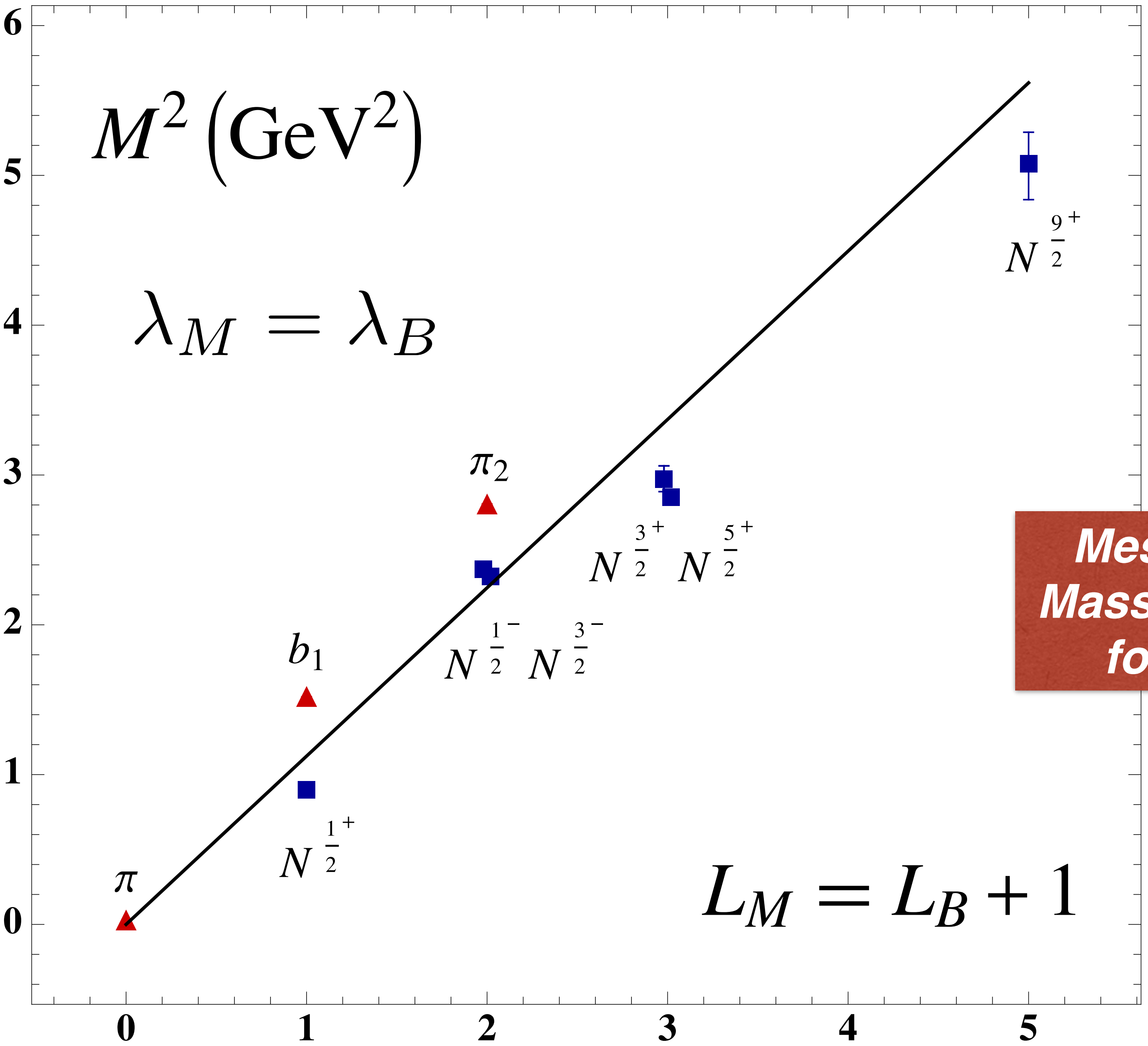
Same κ

$$\lambda = \kappa^2$$

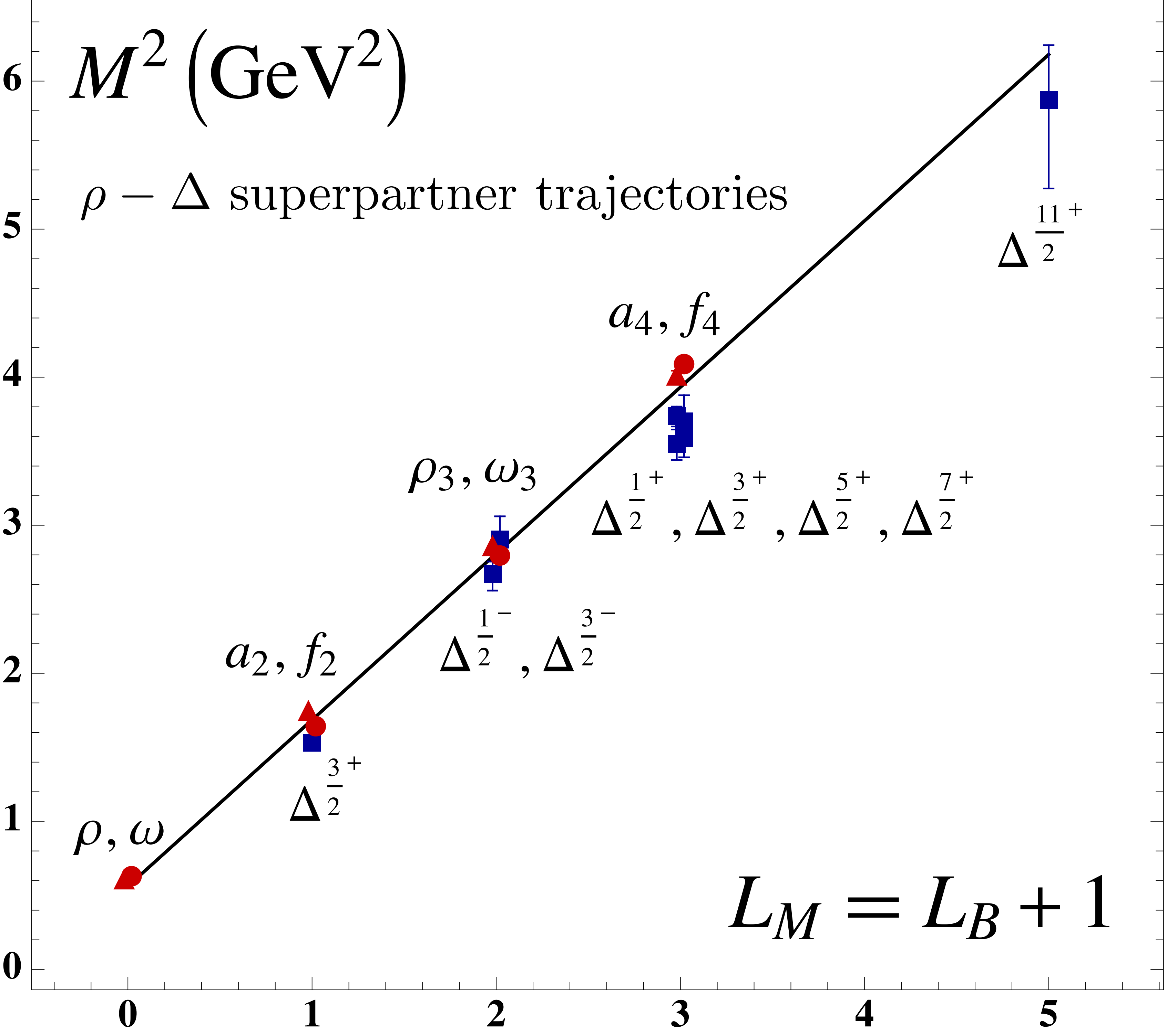
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!

$\lambda = \kappa^2$



Meson-Baryon Mass Degeneracy for $L_M=L_B+1$



Dosch, de Teramond, sjb

Features of Supersymmetric Equations

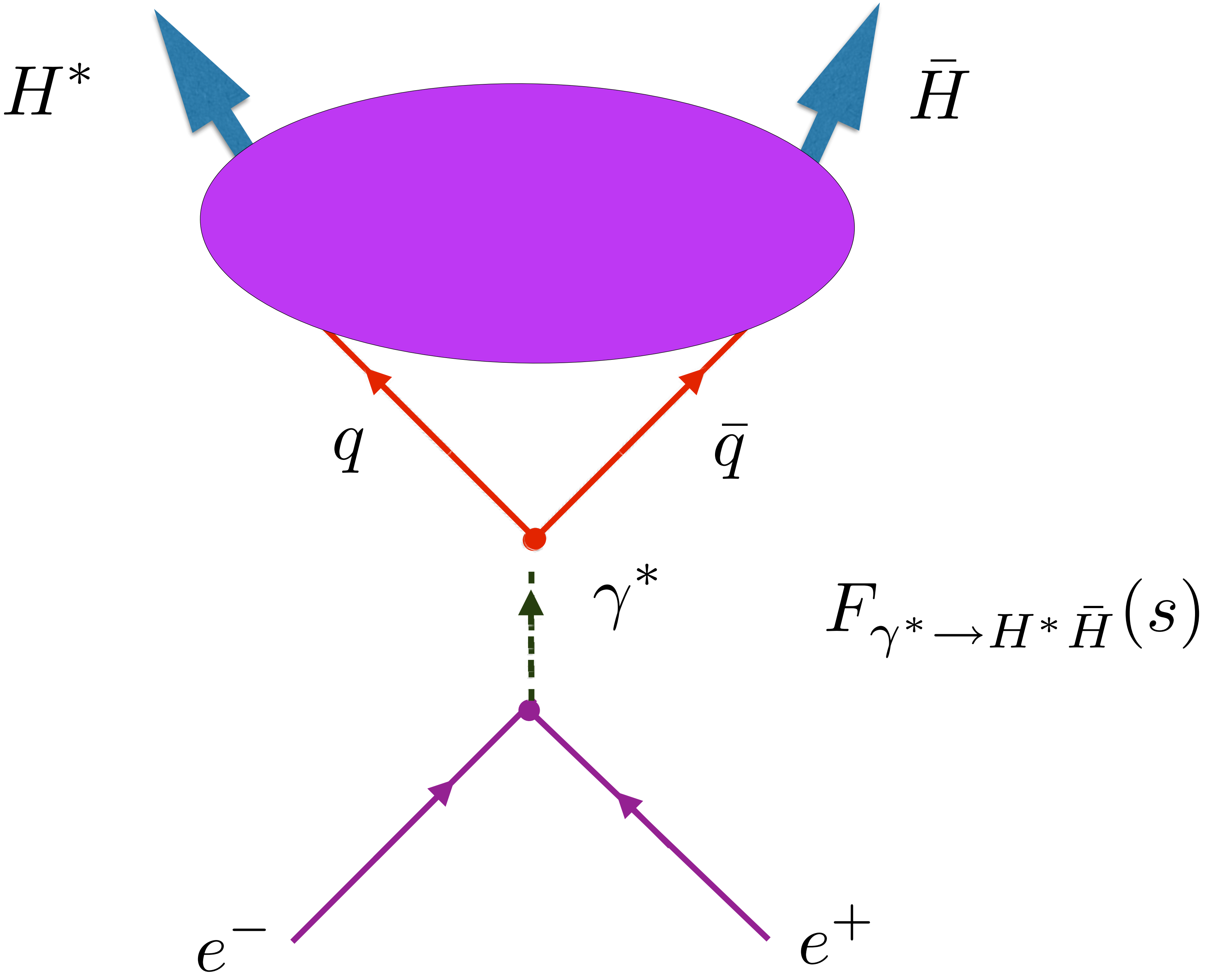
- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $S^z = \pm 1/2$

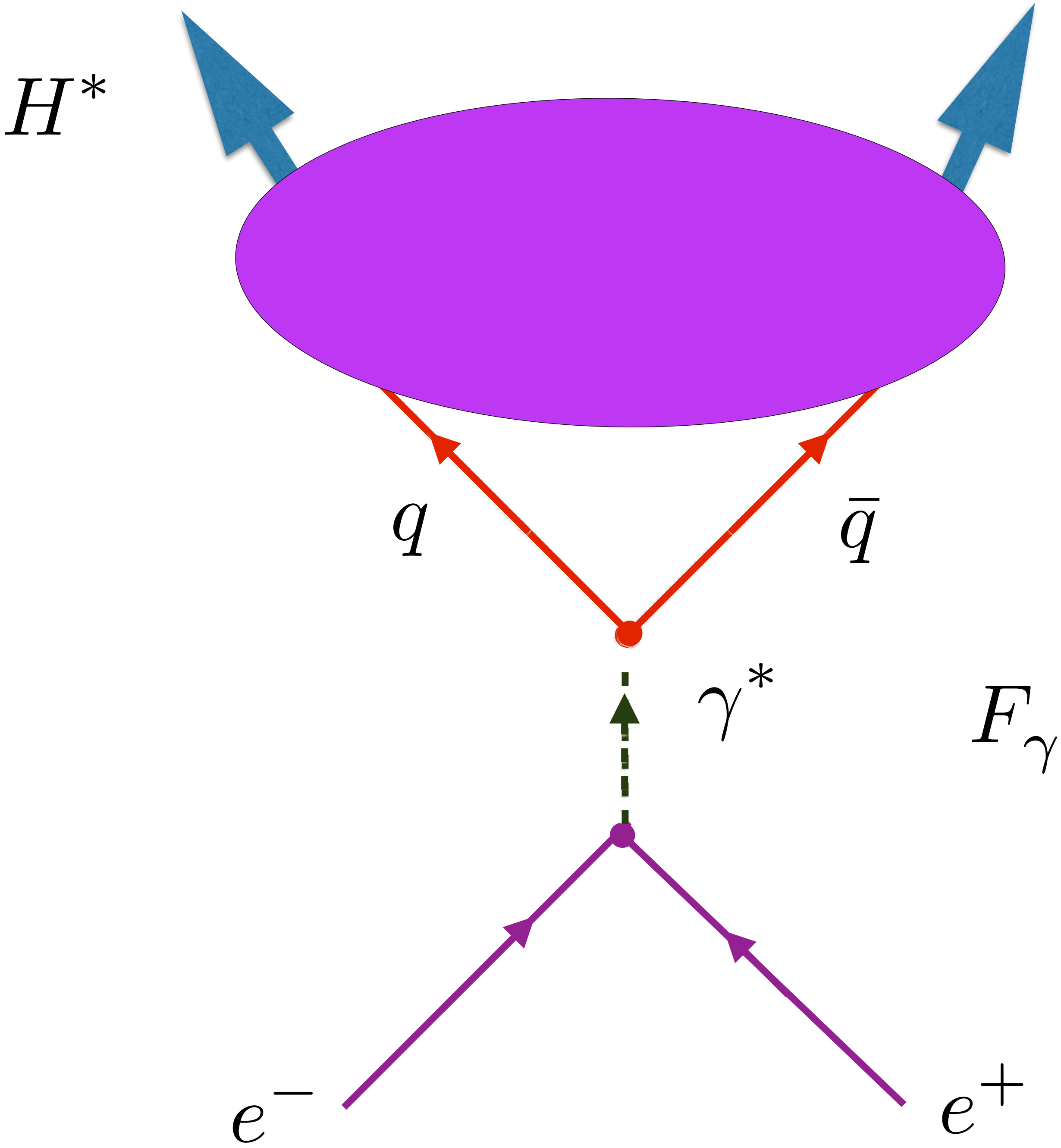
- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$

- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Meson and baryon have same κ !



Timelike Transition Form Factors



Dosch, de Teramond, sjb

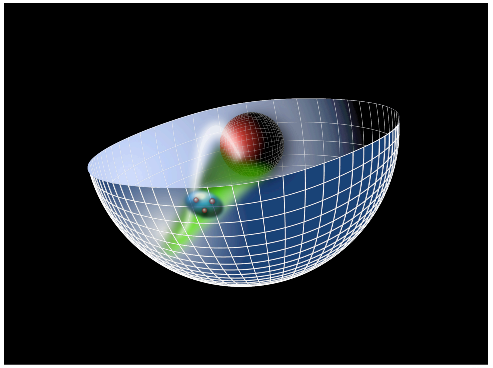
$$F_{\gamma^* \to H^* \bar{H}}(s)$$

Prediction from Super Conformal AdS/QCD:
Same Form Factors for H= M and H=B if $L_M = L_B + 1$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

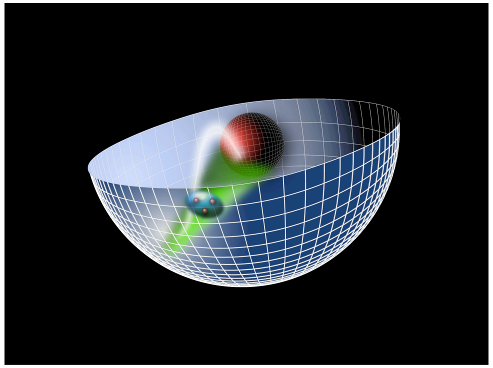
$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal Symmetry
of the action

Light-Front Schrödinger Equation

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π =0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)
Meson-Baryon Mass Degeneracy for L_M=L_B+1

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

- "Zero-Parameter" Model

New Insights into Hadron Physics

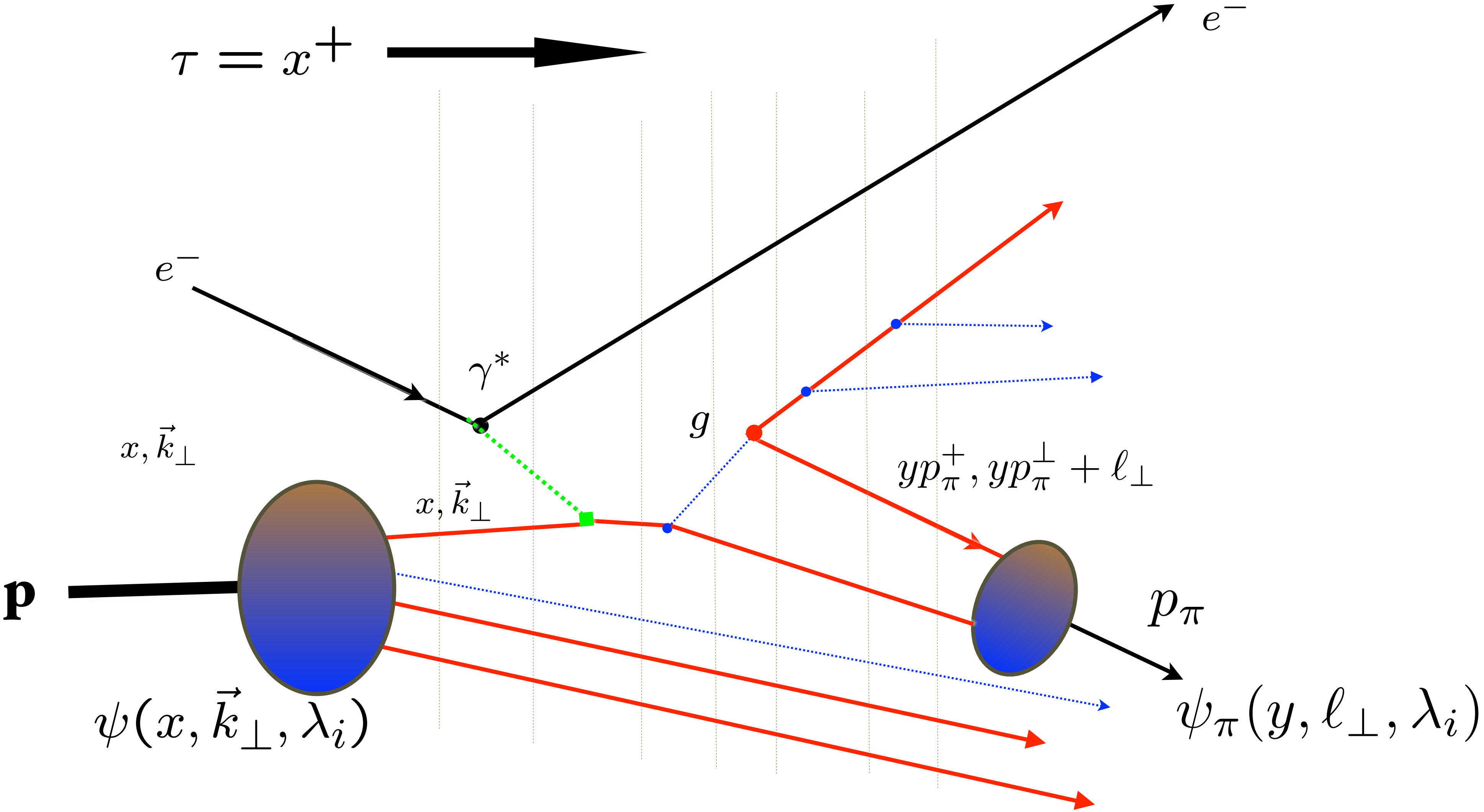
- **Origin of quark confinement?**
- **Determination of the QCD mass scale**
- **Novel hadronic states**
- **Novel QCD phenomena**
- **Supersymmetry in hadron physics**
- **Light-Front Holography**
- **New Physics Opportunities at JLab**

de Tèramond, Dosch, Lorce, sjb

Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**

Hadronization at the Amplitude Level

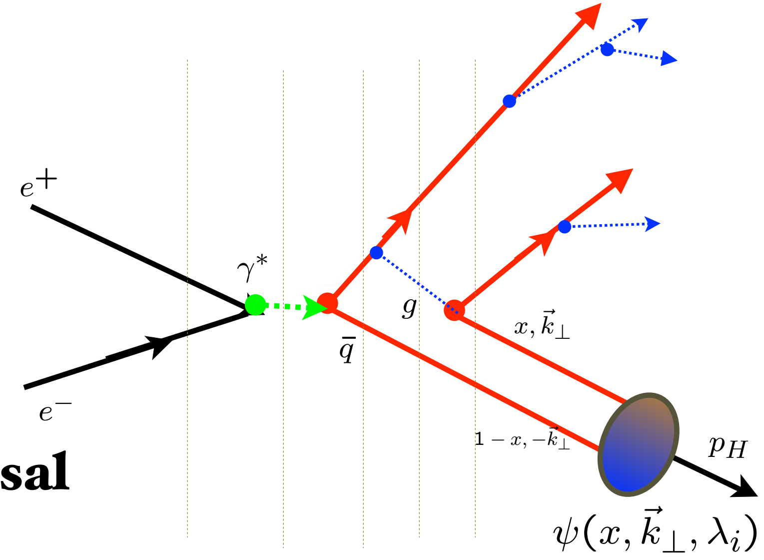


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

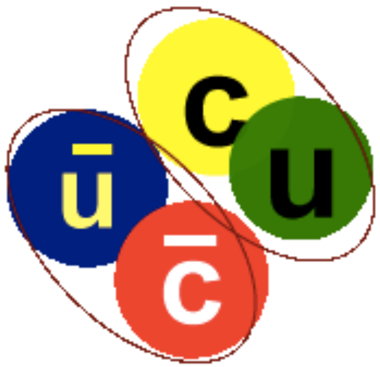
Off -Shell T-Matrix

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** **Chueng Ji, sjb**
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators** Roskies, Suaya, sjb
- **LFWF takes Off-shell to On-shell**

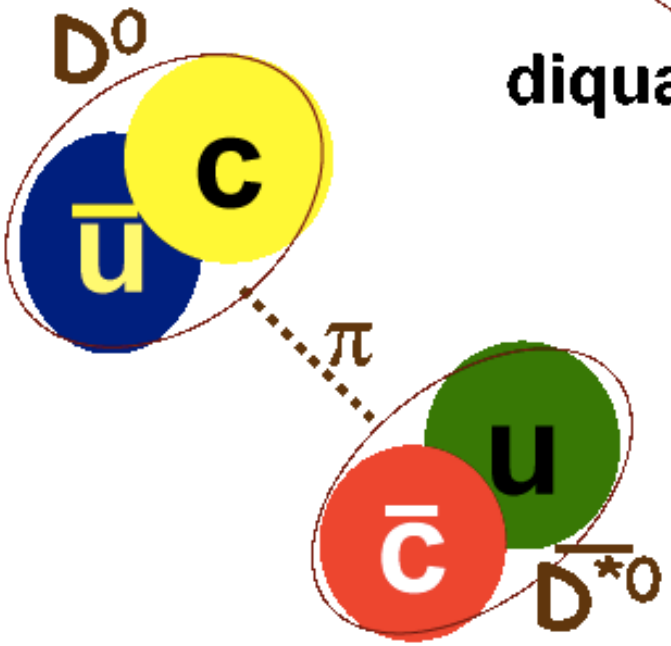


Novel World of Hadron Physics

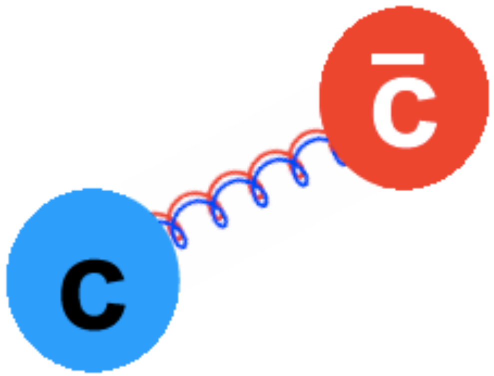
Stan Brodsky



diquark-diantiquark



$D^0 - \overline{D^{*0}}$ “molecule”



$q\bar{q}$-gluon“hybrid”

Four-Quark Hadrons: an Updated Review

arXiv:1411.5997v2

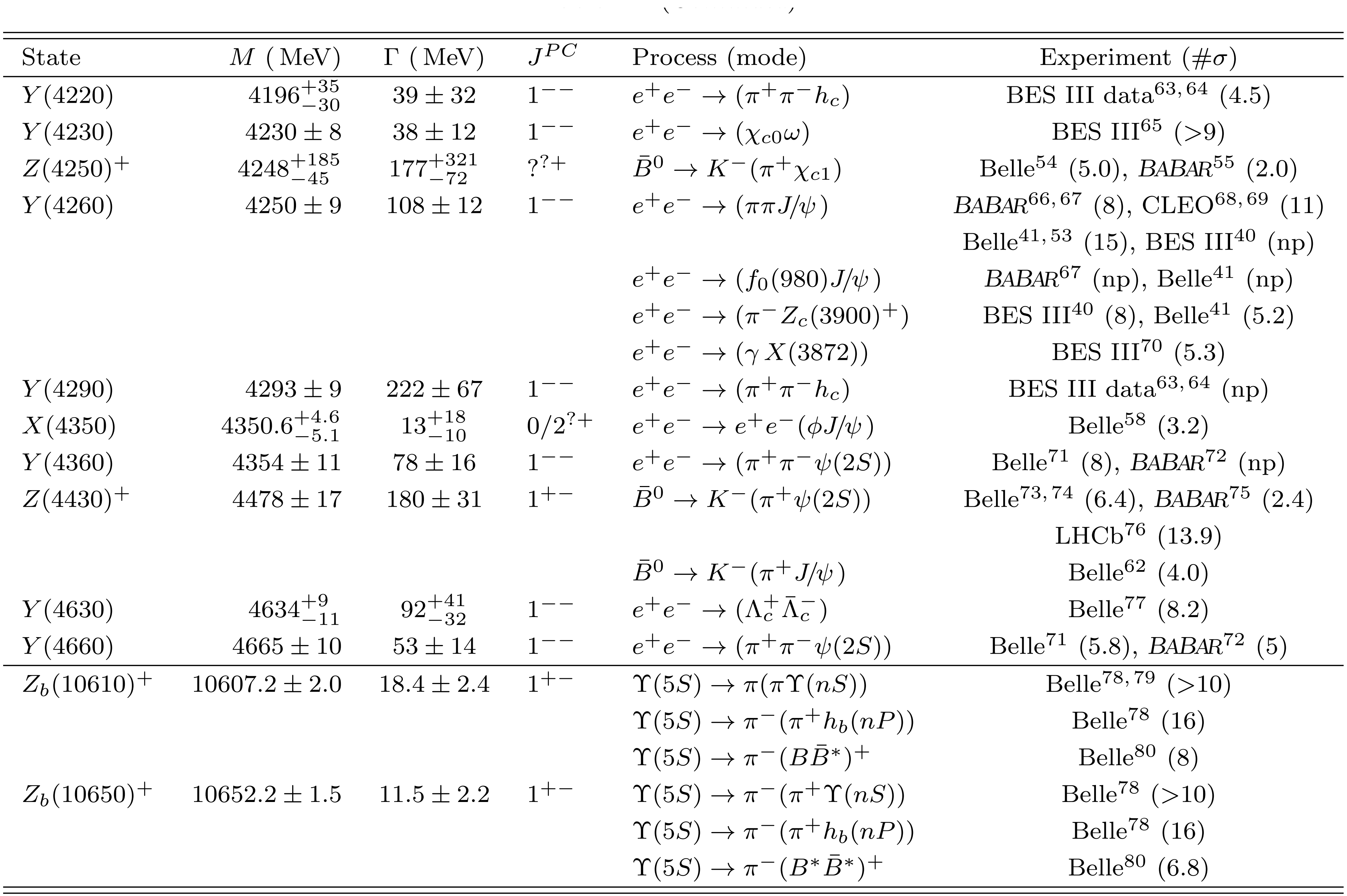
A. ESPOSITOA, L. GUERRIERI, F. PICCININI, A. PILLONI and A. POLOSA

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$B \to K(\chi_{c1}\gamma)$	Belle[23] (4.0)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle[24, 25] (>10), *BABAR*[26] (8.6)
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF[27, 28] (11.6), D0[29] (5.2)
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb[30, 31] (np)
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle[32] (4.3), *BABAR*[33] (4.0)
				$B \to K(\gamma\, J/\psi)$	Belle[34] (5.5), *BABAR*[35] (3.5)
					LHCb[36] (> 10)
				$B \to K(\gamma\, \psi(2S))$	*BABAR*[35] (3.6), Belle[34] (0.2)
					LHCb[36] (4.4)
				$B \to K(D\bar{D}^*)$	Belle[37] (6.4), *BABAR*[38] (4.9)
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III[39] (np)
				$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III[40] (8), Belle[41] (5.2)
					CLEO data[42] (>5)
$Z_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$Y(4260) \to \pi^-(\pi^+ h_c)$	BES III[43] (8.9)
				$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III[44] (10)
$Y(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \to K(\omega J/\psi)$	Belle[45] (8), *BABAR*[33, 46] (19)
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle[47] (7.7), *BABAR*[48] (7.6)
$Z(3930)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle[49] (5.3), *BABAR*[50] (5.8)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\ (D\bar{D}^*)$	Belle[51, 52] (6)
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle[41, 53] (7.4)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (1.1)
$Y(4140)$	4145.6 ± 3.6	14.3 ± 5.9	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF[56, 57] (5.0), Belle[58] (1.9),
					LHCb[59] (1.4), CMS[60] (>5)
					D∅[61] (3.1)
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\ (D^*\bar{D}^*)$	Belle[52] (5.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (7.2)

Four-Quark Hadrons: an Updated Review

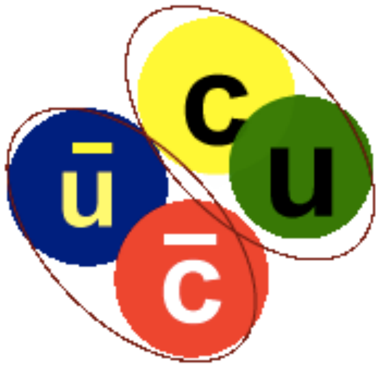
arXiv:1411.5997v2

A. ESPOSITOA, L. GUERRIERI, F. PICCININI, A. PILLONI and A. POLOSA

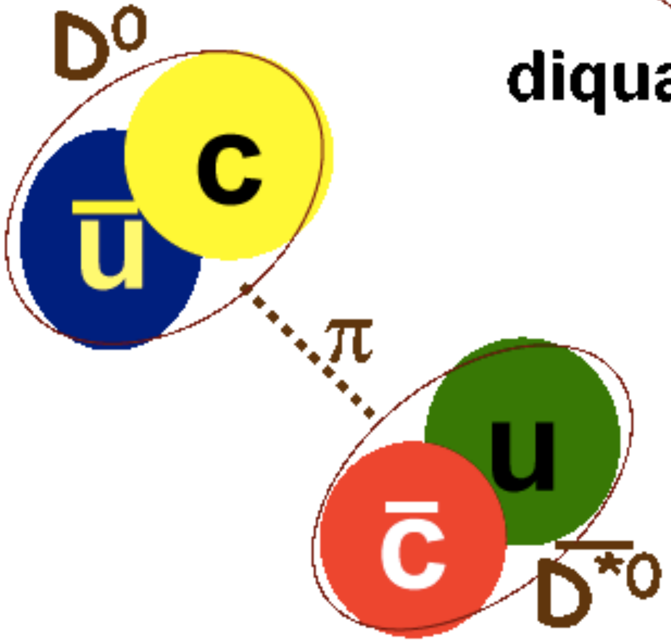


State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$Y(4220)$	4196^{+35}_{-30}	39 ± 32	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63, 64] (4.5)
$Y(4230)$	4230 ± 8	38 ± 12	1^{--}	$e^+e^- \to (\chi_{c0}\omega)$	BES III[65] (>9)
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (2.0)
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	*BABAR*[66, 67] (8), CLEO[68, 69] (11) Belle[41, 53] (15), BES III[40] (np)
				$e^+e^- \to (f_0(980)J/\psi)$	*BABAR*[67] (np), Belle[41] (np)
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III[40] (8), Belle[41] (5.2)
				$e^+e^- \to (\gamma\, X(3872))$	BES III[70] (5.3)
$Y(4290)$	4293 ± 9	222 ± 67	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63, 64] (np)
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle[58] (3.2)
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle[71] (8), *BABAR*[72] (np)
$Z(4430)^+$	4478 ± 17	180 ± 31	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle[73, 74] (6.4), *BABAR*[75] (2.4) LHCb[76] (13.9)
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (4.0)
$Y(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle[77] (8.2)
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle[71] (5.8), *BABAR*[72] (5)
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$	Belle[78, 79] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B\bar{B}^*)^+$	Belle[80] (8)
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(5S) \to \pi^-(\pi^+\Upsilon(nS))$	Belle[78] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B^*\bar{B}^*)^+$	Belle[80] (6.8)

Tetraquarks

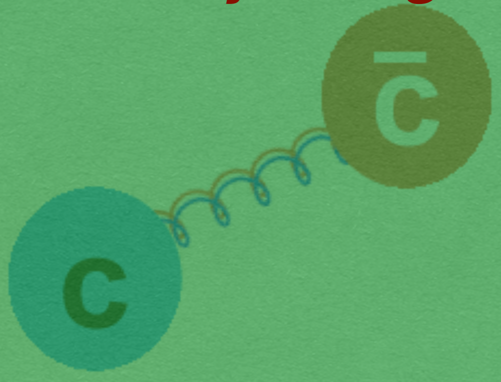


diquark-diantiquark



$D^0 - \overline{D}^{*0}$ “molecule”

Ruled out by charged Z^+_c



$q\bar{q}$-gluon“hybrid”

Belle, BaBar:

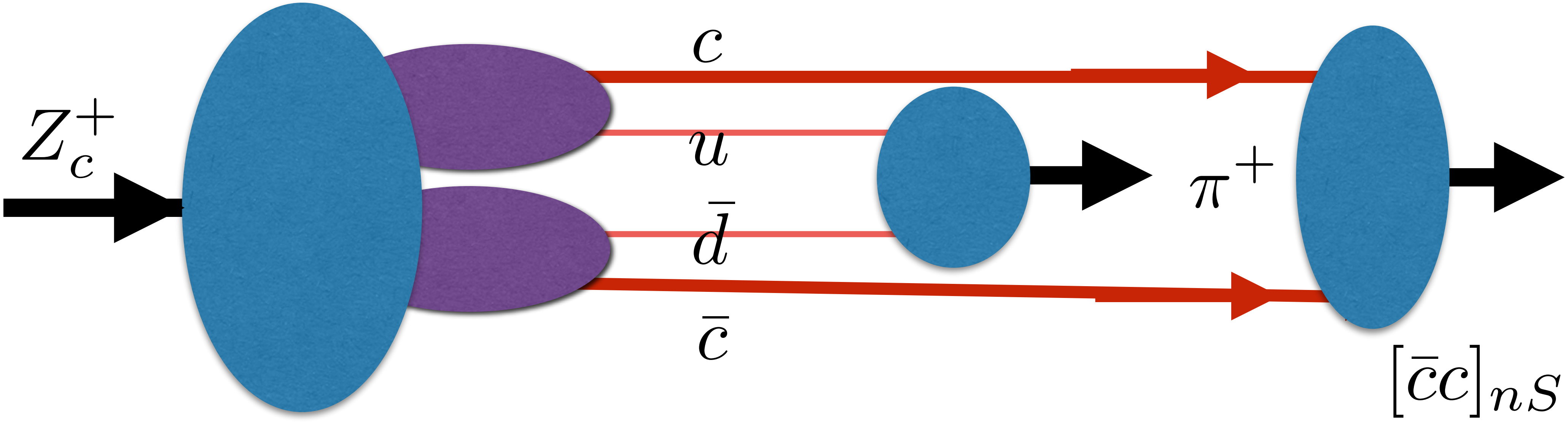
$$\mathcal{B}\left(B^0 \to K^+ Z(4430)^-\right) \times \mathcal{B}\left(Z(4430)^- \to \psi(2S)\pi^-\right) = \left(6.0^{+1.7+2.5}_{-2.0-1.4}\right) \times 10^{-5}.$$

$$\mathcal{B}\left(B^0 \to K^+ Z(4430)^-\right) \times \mathcal{B}\left(Z(4430)^- \to J/\psi\,\pi^-\right) = \left(5.4^{+4.0+1.1}_{-1.0-0.6}\right) \times 10^{-6}.$$

Surprising Result:
Dominance of large-size Ψ' vs J/Ψ decays!

Lebed, Hwang, sjb

Diquark-Diquark



$$Z_c^+([cu][\bar{c}\bar{d}]) \to \pi^+\psi'$$

Formation of charmonium at large separation:

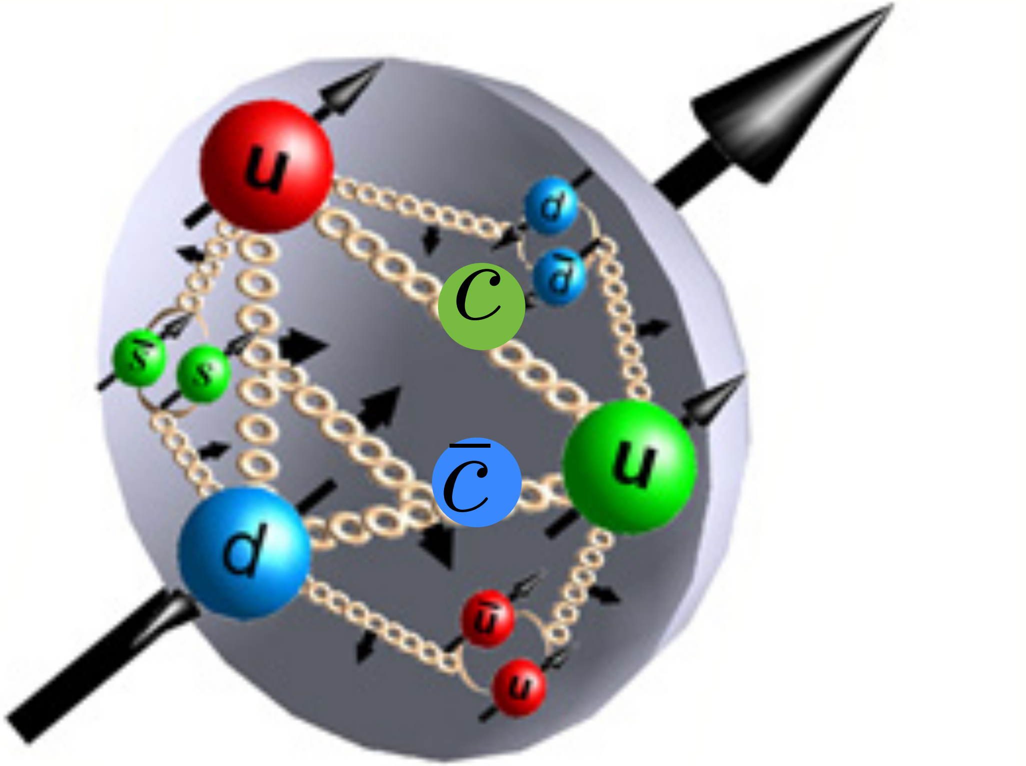
Dominance of overlap with large-size Ψ’ vs J/Ψ decays

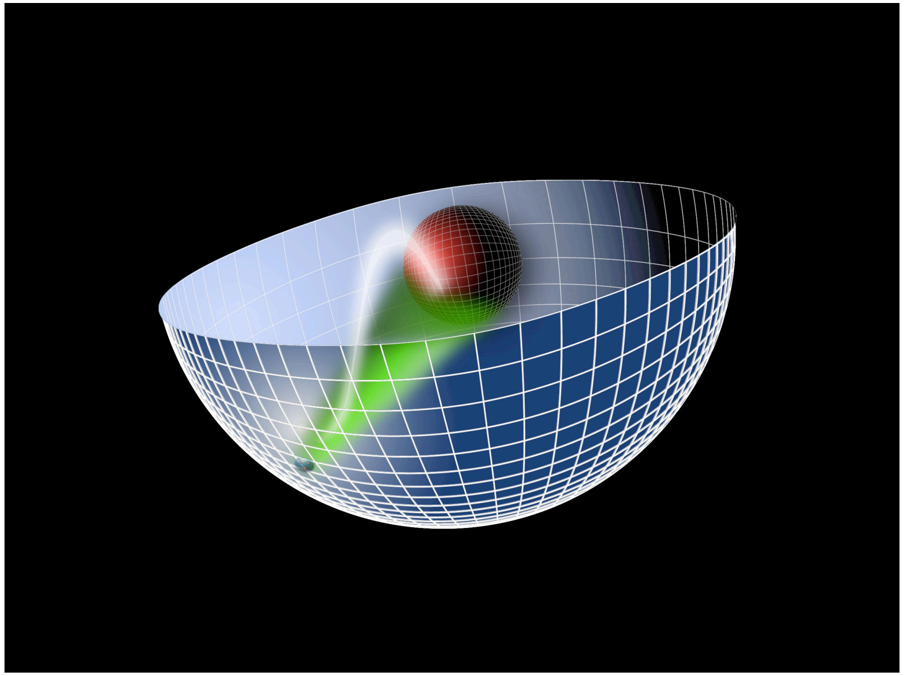
New Opportunities at Jlab

QCD Myths

- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant
- QCD Confinement can only be understood in LGTh
- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale in PQCD cannot be fixed

Novel World of Hadron Physics





Stan Brodsky

Colloquium

April 27, 2015

SLAC NATIONAL ACCELERATOR LABORATORY

UNIVERSITY of VIRGINIA

Stanford University